Chapter 4. Deriving crop yield functions

1. Effects of climate change on crop yields

This model calculates the production of crops, i.e., four gains and two oil crops, by multiplying planted areas by yields. Assessing climate change effects on agricultural production necessitates the use of yield functions.

Climate change is a long-run phenomenon. Increasing temperatures positively affect the lower temperature phase. If temperature exceeds an optimum point, then increasing temperature will negatively affect yields. Therefore, the relation between temperature and yield is expected to represent an inverse U shape.

Furuya and Koyama (2005) estimated macro yield functions using temperature and rainfall data. However, these are specified linearly. The parameters are fixed. To address these shortcomings, they tried to estimate quadratic yield functions considering the dynamic relation between temperature and yield. However, they did not succeed because estimation required data for extremely high temperatures.

To overcome difficulties of relations among yield and climate data, parameters for climate variables are obtained from a crop model (Furuya et al., 2015).

2. Estimation of crop yield trend functions

Recently, crop yields in some countries appear to have hit a ceiling despite widening dissemination of high yield varieties by national and international research institutes (Ray et al., 2012). Considering these circumstances, logistic functions or linear functions with logarithmic time trends are adopted as yield functions.

Removing the effects of climate change in the past, the following functions are estimated as

$$Y_{ik} = a_{ik} + \frac{b_{ik} - a_{ik}}{1 + \exp[-c_{ik}(T - d_{ik})]} + \beta_{TPik}\, TP_{ik} + \beta_{RGik}\, RG_{ik} + \beta_{PTik}\, PT_{ik} + \beta_{GDPik}\, GDPPC_k, \qquad (13\text{-}1)$$

where i is an index of crops, k is an index of countries, Y_{ik} is the yield of crop i, a_{ik} stands for the minimum yield, b_{ik} denotes the maximum yield, c_{ik} represents the slope, d_{ik} is the inflection year, T expresses the time trend, e.g., 1961=1, 1962=2, TP_{ik} signifies the monthly average temperature, RG_{ik} is the monthly average of per-day solar radiation, PT_{ik} stands for monthly rainfall, and $GDPPC_k$ represents the per-capita GDP. In addition, $GDPPC_k$ is a proxy of research investment.

The following linear function is estimated if the fitness of function (13-1) is not good.

$$Y_{ik} = a_{Lik} + b_{Lik} \ln T_L + \beta_{LTPik}\, TP_{ik} + \beta_{LRGik}\, RG_{ik} + \beta_{LPTik}\, PT_{ik} + \beta_{LGDPik}\, GDPPC_k, \qquad (13\text{-}2)$$

Therein, T_L represents the time trend, e.g., 1951=1 and 1952=2.

3. Potential yield of the crop model

Temperature and solar radiation elasticities of yield of the crops are calculated using the parameters of a crop model developed by Doorenbos and Kassam (1979). The functions of the biomass and the yield calculation procedure of their crop model are presented in this section. These are written in their paper.

The net biomass is calculated using the following equation.

$$B_n = B_g - R \qquad (13\text{-}3)$$

Therein, B_n denotes the net biomass production, B_g represents the gross biomass production, and R stands for the respiration loss.

The rate of net biomass production according to the following equation (13-3) as

$$b_n = b_g - r, \qquad (13\text{-}4)$$

where b_n represents the rate of net biomass production, b_g denotes the rate of gross biomass production, and r stands for the respiration rate.

The maximum rate of net biomass production is the rate at which the crop covers the entire ground surface. The point of maximum growth of the net biomass production is the inflection point of the cumulative growth curve. It is assumed that the average rate of net production b_{na} is half of the maximum rate of net biomass production b_{nm}, as in the following model.

$$b_{na} = 0.5\, b_{nm} \qquad (13\text{-}5)$$

Therefore, the net biomass production for a crop of N days is

$$B_n = 0.5\, b_{nm} N. \qquad (13\text{-}6)$$

The maximum rate of gross production b_{gm} depends on the air temperature, the maximum net rate of CO_2 exchange of leaves, the photosynthesis pathway of the crop, and the atmospheric CO_2 concentration. The maximum net rate of CO_2 exchange of leaves P_m and the leaf area index LAI of a standard crop are shown below.

$$P_m = 20 \text{ kg ha}^{-1} \text{ hr}^{-1} \qquad (13\text{-}7)$$

$$LAI = 5 \qquad (13\text{-}8)$$

The maximum rate of gross production b_{gm} is calculated as

$$b_{gm} = F\, b_o + (1 - F)\, b_c, \qquad (13\text{-}9)$$

where F is the rate of covered by cloud of the sky in the daytime, as calculated using the following equation.

$$F = (A_c - 0.5\, R_g) / (0.8\, A_c) \qquad (13\text{-}10)$$

In that equation, A_c is the maximum active incoming short-wave radiation on a clear day. In addition, R_g is the incoming short-wave radiation. The units of both variables are cal cm^{-2} day^{-1}.

The b_o and the b_c in equation (13-9) are the rates of gross dry matter production of a crop, respectively, on a completely overcast day and on a perfectly clear day. The units of these variables are kg ha^{-1} day^{-1}.

The maximum rate of gross production b_{gm} is calculated using the equation below if the maximum net rate of CO_2 exchange of leaves P_m exceeds 20 kg ha^{-1} hr^{-1}.

$$b_{gm} = F\,(0.8 + 0.01\,P_m)\,b_o + (1 - F)\,(0.5 + 0.025\,P_m)\,b_c \qquad (13\text{-}11)$$

The maximum rate of gross production b_{gm} is calculated using the following equation if the maximum net rate of CO_2 exchange of leaves P_m is less than 20 kg ha^{-1} hr^{-1}:

$$b_{gm} = F\,(0.5 + 0.025\,P_m)\,b_o + (1 - F)\,(0.05\,P_m)\,b_c \qquad (13\text{-}12)$$

The maximum rate of gross production b_{gm} and the maximum respiration rate r_m are necessary for obtaining the maximum rate of net production b_{nm}. The maximum respiration rate r_m is calculated as

$$r_m = k\,b_{gm} + c_t\,B_m, \qquad (13\text{-}13)$$

where k is a proportional constant and c_t represents the relative maintenance respiration rate. In addition, B_m is the accumulated net biomass when the net biomass production rate is the maximum. Herein, the k of beans and other crops are 0.28:

$$k = 0.28\,. \qquad (13\text{-}14)$$

However, the relative maintenance respiration rate c_t depends on temperature: it takes different numbers in beans and other crops. The following numbers are taken at 30°C for beans and other crops.

$$c_{30i} = 0.0283 \text{ for } i\text{: } SB, XS \qquad (13\text{-}15)$$

$$c_{30i} = 0.0108 \text{ for } i\text{: } RI, WH, MZ, XG \qquad (13\text{-}16)$$

The relative maintenance respiration rate c_t is obtained using the following function.

$$c_t = c_{30i}\,(0.0044 + 0.0019t + 0.0010t^2) \qquad (13\text{-}17)$$

The accumulated net biomass B_m is assumed as half of the net biomass production B_n, as shown below.

$$B_m = 0.5\,B_n \qquad (13\text{-}18)$$

Substituting equation (13-6) into equation (13-18) gives the equation shown below.

$$B_m = 0.25\,b_{nm}\,N \qquad (13\text{-}19)$$

If the equation of the rate of net biomass production (13-4) is rewritten for the maximum rate of net production b_{nm}, the maximum rate of gross production b_{gm}, and the maximum respiration rate r_m, then the following equation is obtained.

$$b_{nm} = b_{gm} - r_m \qquad (13\text{-}20)$$

Substituting equation (13-13) into equation (13-20) yields the following equation.

$$b_{nm} = b_{gm} - k\,b_{gm} - c_t\,B_m = (1 - k)\,b_{gm} - c_t\,B_m \qquad (13\text{-}21)$$

Substituting equation (13-19) into equation (13-21) gives the equation shown below.

$$b_{nm} = (1 - k)\,b_{gm} - c_t\,(0.25\,b_{nm}\,N)$$

$$(1 + 0.25\,c_t\,N)\,b_{nm} = (1 - k)\,b_{gm}$$

$$b_{nm} = (1 - k)\,b_{gm} / (1 + 0.25 c_t\,N) \qquad (13\text{-}22)$$

Substituting the constant k (13-14) into equation (13-22) gives the equation shown below.

$$b_{nm} = 0.72\,b_{gm} / (1 + 0.25\,c_t\,N) \qquad (13\text{-}22)$$

By substituting equation (13-22) into equation (13-6), the equation of the net biomass production for a crop B_n is obtained. It is explained as the maximum rate of gross production b_{gm} in the case of the LAI is equal to five, days of the growing period N, and the relative maintenance respiration rate c_t .

$$B_n = 0.5\,[0.72\,b_{gm} / (1 + 0.25\,c_t\,N)]\,N = 0.36\,b_{gm} / (1/N + 0.25c_t) \qquad (13\text{-}23)$$

Equation (13-23) shows the net biomass production for a crop B_n for LAI=5. However, the real net biomass production for a crop B_n is obtained by multiplying L, which is the rate of the actual LAI to the LAI=5. The following equation shows the real net biomass production.

$$B_n = 0.36\,b_{gm}\,L / (1/N + 0.25c_t) \qquad (13\text{-}24)$$

Potential yield Y_p is obtained from the real net biomass production for a crop B_n, and the harvest index HI, which is the rate of a biomass of an economically useful crop to the net biomass of the crop.

$$Y_p = HI\,B_n \qquad (13\text{-}25)$$

The required variables for estimation of the potential yield are presented below.

(a) A_c: the maximum active incoming short-wave radiation on a clear day
(b) R_g: Incoming short-wave radiation
(c) N: Days of the growing period, i.e., from germination to ripeness
(d) P_m: Maximum net rate of CO_2 exchange of leaves
(e) L: Rate of the actual LAI to the LAI=5
(f) HI: Harvest index

4. Example of potential yield calculation

The following example can be calculated.
Crop: rice, Japonica, wetland
Growth cycle: N=135 days (May−September)
Input: intermediate
Crop group: C3/II
Harvest index, HI: 0.35
LAI: 4.3, L=4.3/5=0.86
P_m: the maximum net rate of CO_2 exchange of leaves (kg ha^{-1} hr^{-1}): 0 (5°C), 5 (10°C), 15 (15°C), 30 (20°C), 35 (25°C), 35 (30°C), 30 (35°C), 5 (40°C), and 0 (45°C)

The linear approximation function of the P_m to the temperature band is shown below.

$$P_m = -5 + t \;(5\text{−}10°\text{C})$$

$$P_m = -5 + 2t \;(10\text{−}15°\text{C})$$

$$P_m = -30 + 3t \;(15\text{−}20°\text{C})$$

$$P_m = +10 + t \;(20\text{−}25°\text{C})$$

$P_m = +\,35$ (25–30°C)
$P_m = +\,65 - t$ (30–35°C)
Therein, t represents the air temperature.
R_g: Incoming short-wave radiation in Niigata, Japan in 1997 (cal cm^{-2} day^{-1})
R_g: Average from May–September, 398.8
(16.69 MJ m^{-2} day^{-1})
A_c: Maximum active incoming short-wave radiation on a clear day (cal cm^{-2} day^{-1}) is the following (Doorenbos and Kassam (1979), FAO Irrigation & Drainage Paper 33, P9):
A_c: Average of May–September, 40°, 379.6
F: The rates of coverage of the sky by clouds during the daytime of these months are shown below.
$F = (A_c - 0.5\,R_g) / (0.8\,A_c)$
F: Average of May–September.
$= (379.6 - 0.5 \times 398.8)/(0.8 \times 379.6) = 0.593$
b_o: Rates of gross dry matter production of a crop on a completely overcast day (kg ha^{-1} day^{-1})
b_o: Average of May–September, 40°, 244.6
b_c: the rates of gross dry matter production of a crop on a perfectly clear day (kg ha^{-1} day^{-1})
b_c: Average of May–September, 40°, 465.6
In this case, the potential yield is the following.

$$\begin{aligned} Y_p &= HI\,B_n \\ &= 0.35\,B_n \\ &= 0.35\,[0.36\,b_{gm}\,L / (1/N + 0.25\,c_t)] \\ &= 0.35\,[0.36\,b_{gm}\,0.86 / (1/135 + 0.25\,c_t)] \\ &= 0.11\,b_{gm} / (0.0074 + 0.25\,c_t) \end{aligned} \tag{13-26}$$

One can derive the potential yield for temperature t of 20–25°C. In this case, the maximum net rate of CO_2 exchange of leaves is

$P_m = 10 + t$.

The maximum rate of gross production is calculated using equation (13-11) because P_m exceeds 20. If $P_m = 10{+}t$, $F = 0.593$, $b_o = 244.6$, $b_c = 465.6$ are substituted in equation (13-11), then the following function is obtained.

$$\begin{aligned} b_{gm} &= F\,(0.8 + 0.01\,P_m)\,b_o \\ &\quad + (1 - F)\,(0.5 + 0.025\,P_m)\,b_c \\ &= 0.593\,[0.8 + 0.01\,(10 + t)] \times 244.6 \\ &\quad + (1 - 0.593)\,[0.5 + 0.025\,(10 + t)] \times 465.6 \\ &= 272.667 + 6.188t \end{aligned} \tag{13-27}$$

By substituting equation (13-27) into (13-26), the following function of the potential yield is obtained.

$$Y_p = \frac{29.546 + 0.671t}{0.0074 + 0.25c_t} \tag{13-28}$$

$$c_t = 0.00004752 + 0.00002052t + 0.00001080t^2 \tag{13-29}$$

The potential yield will be the following if the temperature is 22°C.

$c_t = 0.009789$

$$Y_p = \frac{29.546 + 14.762}{0.0074 + 0.00245} = 4{,}498 \text{ (kg ha}^{-1}\text{)}$$

5. Yield function with variable climate parameters using the crop model

(1) Smoothing functions of the crop model

Some functions of the model indicate dramatic changes in section 3 at some points. In these cases, the estimated yield will bend around the points of the climate variable. To prevent dramatic changes in the estimated yield, the two functions are smoothed using the logistic curve and spline interpolation.

1) Maximum rate of biomass production

In the original model, if the maximum net rate of CO_2 exchange of leaves (P_m) is less than 20 kg ha^{-1} hr^{-1}, then the function of the maximum rate of gross production (b_{gm}) is (13-12). If P_m exceeds 20 kg ha^{-1} hr^{-1}, then the function of b_{gm} is (13-11).

The system of these functions is changed as follows:
The function of b_{gm} will be (13-12) if P_m <15.
The function of b_{gm} will be (13-11) if P_m≧25.
The function of b_{gm} will be the following function if 15≦ P_m < 25.

$$\begin{aligned} b_{gm} = F&\left[\left(0.5 + \frac{0.3}{1+e^{20-pm}}\right)\right. \\ &\left. +\left(0.01 + \frac{0.015}{1+e^{pm-20}}\right)P_m\right]b_0 \\ &+(1-F)\left[\left(\frac{0.5}{1+e^{20-pm}}\right)\right. \\ &\left. +\left(0.025 + \frac{0.025}{1+e^{pm-20}}\right)P_m\right]b_c \end{aligned} \tag{13-30}$$

Figure 4-1 shows the relation between P_m and b_{gm} for F=0.6, b_o=250, and b_c=450.

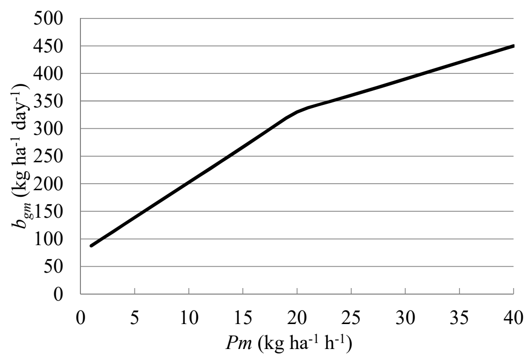


Figure 4-1. Relation between the maximum net CO_2 exchange rate (P_m) and maximum gross biomass production rate (b_{gm}).

2) Maximum net CO_2 exchange rate of leaves

The maximum net rates of CO_2 exchange of leaves (P_m) are provided in a report of Fischer et al. (2002). However, these numbers are point data presented for 5°C increments. Therefore, P_m will change dramatically around temperatures given in multiples of five.

Alleviating the dramatic changes in thresholds, the cubic-spline (CS) interpolations are applied to the P_m data. If the points of the data are (x_1,y_1), (x_2,y_2), $\cdots$, (x_n,y_n) for $(a \le x_1 \le x_2 \le \cdots \le x_n \le b)$ and if the function $f(x_i)=y_i$, for $i=1,2,\cdots,n$ is continuous and differentiable, then the CS function of $[x_i\, x_{i+1}]$ is defined as shown below.

$$S_i(x)=a_i+b_i(x-x_i)+c_i(x-x_i)^2+d_i(x-x_i)^3 \quad (i=1,2,\ \cdots\ ,n-1) \qquad (13\text{-}31)$$

Using conditions of interpolation and continuity of the first and second derivatives on the tangent points, parameters c_i are obtained by solving the tri-diagonal matrix function; other parameters are obtained from the conditions of continuities (Shimoda and Tabe, 1990).

Figure 4-2 presents the relation between temperature and P_m of Japonica rice in a wetland area.

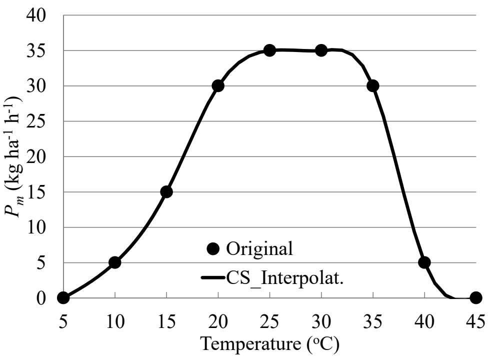


Figure 4-2. Relation between temperature and maximum net CO_2 exchange rate (P_m) of rice, Japonica, wetland.

(2) Derivation of temperature elasticities of the potential yield

The temperature elasticity of the potential yield is calculated as

$$\frac{\partial \ln Y_p}{\partial \ln TP}=\frac{\partial Y_p}{\partial TP}\frac{TP}{Y_p}=\frac{\partial B_n}{\partial TP}\frac{TP}{B_n}$$

$$=\frac{\partial b_{gm}}{\partial TP}\frac{TP}{b_{gm}}$$

$$+\frac{\partial\left(1/N+0.25c_t\right)^{-1}}{\partial TP}TP\left(1/N+0.25c_t\right) \qquad (13\text{-}32)$$

where Y_p stands for the potential yield (kg ha^{-1}), TP denotes the temperature (°C), B_n expresses the rate of net biomass production (kg ha^{-1}), b_{gm} signifies the maximum rate of gross biomass production (kg ha^{-1} day^{-1}), N represents the total growing days (day), and c_t is a constant proportion of maintenance respiration (g g^{-1} day^{-1}.)

The potential yield is calculated using equation (13-25). The gap separating the potential and actual yields is explained by evapotranspiration in the model of Doorenbos and Kassam (1979). The total growing days (N) are estimated from the cropping calendar of the USDA (1994).

Substituting $\partial b_{gm}/\partial TP$, i.e., the marginal propensity of the maximum rate of gross biomass production to temperature, and c_t, i.e., a constant proportion of maintenance respiration (13-17), into equation (13-32), the temperature elasticities of potential yield are obtained as expressed below.

If $P_m < 15$, then

$$\frac{\partial \ln Y_p}{\partial \ln TP}=\frac{\left[0.025Fb_o+0.05(1-F)b_c\right]TP}{b_{gm}}\frac{\partial P_m}{\partial TP}$$

$$-\frac{0.25(0.0019+0.0020TP)c_{30}TP}{1/N+0.25c_t} \qquad (13\text{-}33)$$

If $15 \le P_m < 25$, then

$$\frac{\partial \ln Y_p}{\partial \ln TP}=\left[\frac{0.3Fb_o+0.5(1-F)b_c}{(1+e^{20-Pm})^2}e^{20-Pm}\right.$$

$$+(0.01F\,b_o+0.025(1-F)bc$$

$$\left.+\frac{0.015Fb_0+0.025(1-F)b_c}{1+e^{Pm-20}}\right)$$

$$\left.-\frac{0.015Fb_0+0.025(1-F)b_c}{(1+e^{Pm-20})^2}e^{Pm-20}P_m\right]$$

$$\times\frac{\partial P_m}{\partial t}\frac{TP}{b_{gm}}$$

$$-\frac{0.25(0.0019+0.0020TP)c_{30}TP}{1/N+0.25c_t} \qquad (13\text{-}34)$$

If $P_m \ge 25$, then

$$\frac{\partial \ln Y_p}{\partial \ln TP}=\frac{\left[0.01Fb_o+0.025(1-F)b_c\right]TP}{b_{gm}}\frac{\partial P_m}{\partial TP}$$

$$-\frac{0.25(0.0019+0.0020TP)c_{30}TP}{1/N+0.25c_t} \qquad (13\text{-}35)$$

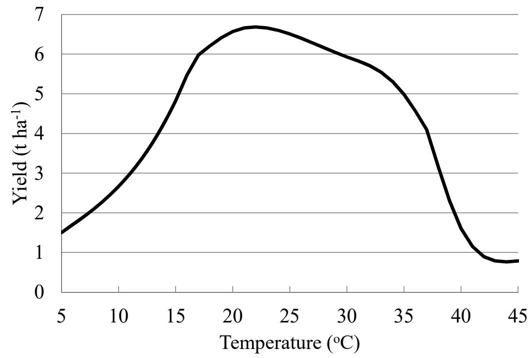
However, the potential yield is obtained from equations (13-24) and (13-25). Substituting equation (13-24) into equation (13-25) gives the equation presented below.

$$Y_p = \frac{0.36HI \cdot b_{gm} \cdot LAI / 5}{1/N + 0.25c_t} \tag{13-36}$$

Figure 4-3 presents relations between the temperature and potential yield of Japonica rice in wetland, winter wheat, maize in sub-tropics, and soybeans in the tropics of the crop model of Doorenbos and Kassam (1979).

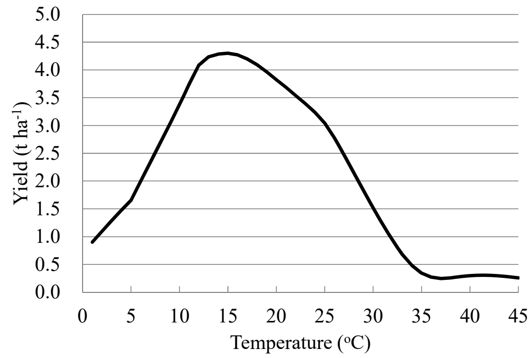
These graphs show smoothing loci based on the modified functions of the maximum rate of gross biomass production and the maximum net rates of CO_2 exchange of leaves, as shown in Figures 4-1 and 4-2.

Total growing days (N) (day), harvest index (HI) (dimensionless number), leaf area index (LAI) (dimensionless number), the gross dry matter production rate on a completely overcast day and on a perfectly clear day (b_o, b_c) (kg ha^{-1} da^{-1}), and solar radiation (RG) (MJ m^{-2} day^{-1}) are shown in the graph notation. The unit of solar radiation is changed from cal cm^{-2} day^{-1} to MJ m^{-2} day^{-1} in these graphs of Figure 4-3 (1 MJ m^{-2} day^{-1} = 23.89 cal cm^{-2} day^{-1}).



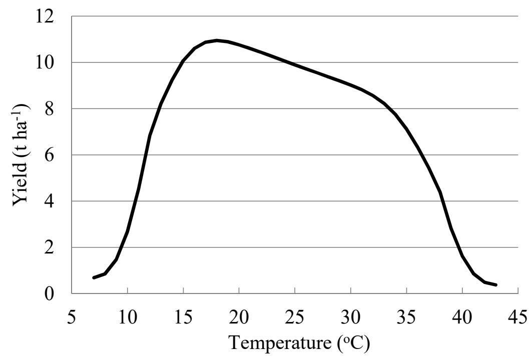
(i) Japonica rice in wetland

N=165, HI=0.3, LAI=6.0, bo=231, bc=442, RG=15 (MJ m^{-2} day^{-1})



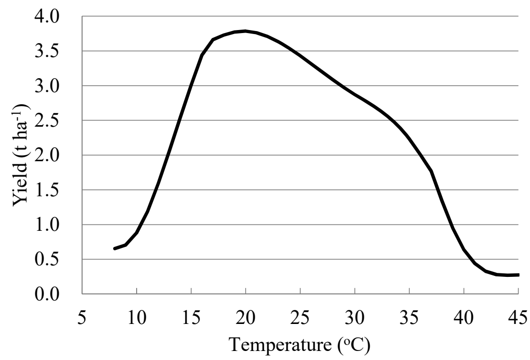
(ii) Winter wheat

N=300, HI=0.2, LAI=4.0, bo=178, bc=353, RG=14 (MJ m^{-2} day^{-1})



(iii) Maize in sub-tropics

N=165, HI=0.45, LAI=4.5, bo=216, bc=417, RG=18 (MJ m^{-2} day^{-1})



(iv) Soybeans in tropics

N=185, HI=0.3, LAI=4.0, bo=232, bc=434, RG=17 (MJ m^{-2} day^{-1})

Figure 4-3. Relation between temperature and potential yield.

(3) Derivation of solar-radiation elasticities of the potential yield

The solar-radiation elasticity of potential yield is calculated using the following equation.

$$\frac{\partial \ln Y_p}{\partial \ln RG} = \frac{\partial Y_p}{\partial RG}\frac{RG}{Y_p}$$

$$= \frac{\partial B_n}{\partial RG}\frac{RG}{B_n} = \frac{\partial b_{gm}}{\partial RG}\frac{RG}{b_{gm}}$$

$$= \frac{\partial b_{gm}}{\partial F}\frac{\partial F}{\partial RG}\frac{RG}{b_{gm}} \tag{13-37}$$

The marginal propensity of F to RG is shown below.

$$\frac{\partial F}{\partial RG} = -\frac{0.625}{A_c} \tag{13-38}$$

By substituting the marginal propensity of b_{gm} to F and that of F to RG (13-38) into equation (13-37), the solar radiation elasticities of the potential yield are obtained as presented below.

If $P_m < 15$, then

$$\frac{\partial \ln Yp}{\partial \ln RG} = -\frac{0.625}{A_c}[(0.5 + 0.025P_m)b_o$$

$$-0.05\,P_m\,b_c]\,(RG/b_{gm}) \qquad (13\text{-}39)$$

If $15 \leq P_m < 25$, then

$$\frac{\partial \ln Yp}{\partial \ln RG} = -\frac{0.625}{A_c}[0.05b_0 + 0.75b_c$$

$$+(0.0775\,b_o - 0.1375\,b_c)\,P_m$$

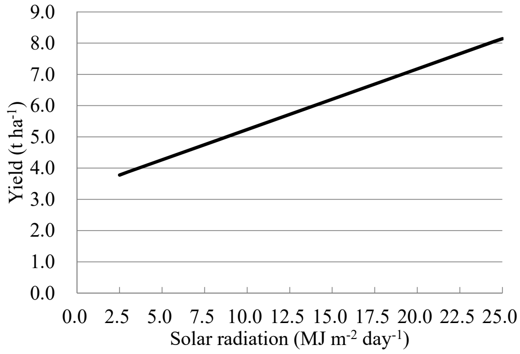
$$-(0.0015b_o - 0.0025b_c)P_m^{2}]\,(RG/b_{gm}). \qquad (13\text{-}40)$$

If $P_m \geq 25$, then

$$\frac{\partial \ln Yp}{\partial \ln RG} = -\frac{0.625}{A_c}[(0.8 + 0.01P_m)b_o$$

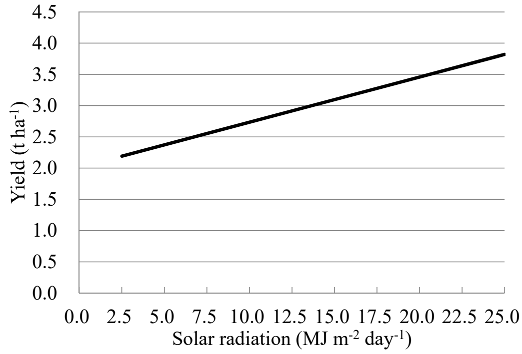
$$-(0.5 + 0.025\,P_m)\,b_c]\,(RG/b_{gm}). \qquad (13\text{-}41)$$

Figure 4-4 presents the relations between solar radiation and the potential yield of the crop model of Doorenbos and Kassam (1979) for given conditions. *N*, *HI*, *LAI*, b_o, b_c, and *TP* are shown in the graph notation.



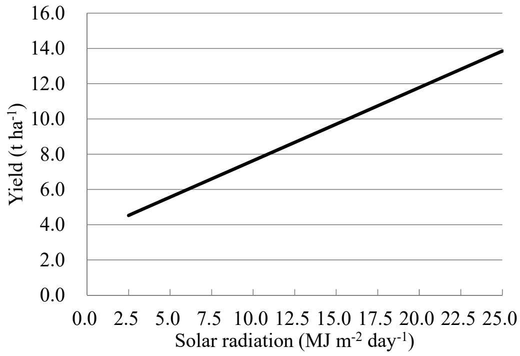
(i) Japonica rice in wetland

N=165, *HI*=0.3, *LAI*=6.0, *bo*=231, *bc*=442, *TP*=18



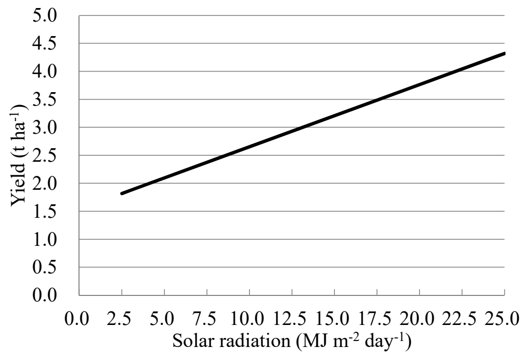
(ii) Winter wheat

N=300, *HI*=0.2, *LAI*=4.0, *bo*=178, *bc*=353, *TP*=9



(iii) Maize in sub-tropics

N=165, *HI*=0.45, *LAI*=4.5, *bo*=216, *bc*=417, *TP*=18



(iv) Soybeans in tropics

N=185, *HI*=0.3, *LAI*=4.0, *bo*=232, *bc*=434, *TP*=25

Figure 4-4. Relation between solar radiation and potential yield.

(4) Incorporating temperature and solar-radiation elasticities into the yield functions

Yield functions specified as logistic functions with marginal propensity to temperature, solar radiation, and rainfall in the base year *tB* and the next year *tB*+1 are shown below.

$$Y_{iktB} = a_{ik} + \frac{b_{ik} - a_{ik}}{1 + \exp[-c_{ik}(T_{tB} - d_{ik})]}$$

$$+\frac{\partial Y_{piktB}}{\partial TP_{iktB}}TP_{iktB} + \frac{\partial Y_{piktB}}{\partial RG_{iktB}}RG_{iktB} + \frac{\partial Y_{pik}}{\partial PT_{ik}}PT_{iktB}$$

$$+\frac{\partial Y_{pik}}{\partial GDPPC_k}GDPPC_{ktB} \qquad (13\text{-}42)$$

$$Y_{iktB+1} = a_{ik} + \frac{b_{ik} - a_{ik}}{1 + \exp[-c_{ik}(T_{tB+1} - d_{ik})]}$$

$$+\frac{\partial Y_{piktB}}{\partial TP_{iktB}}TP_{lktB} + \frac{\partial Y_{piktB}}{\partial RG_{iktB}}RG_{iktB} + \frac{\partial Y_{pik}}{\partial PT_{ik}}PT_{iktB}$$

$$+\frac{\partial Y_{pik}}{\partial GDPPC_k}GDPPC_{ktB}$$

$$+\frac{1}{2}\left(\frac{\partial Y_{piktB+1}}{\partial TP_{iktB+1}} + \frac{\partial Y_{piktB}}{\partial TP_{iktB}}\right)(TP_{iktB+1} - TP_{iktB})$$

$$+\frac{1}{2}\left(\frac{\partial Y_{piktB+1}}{\partial RG_{iktB+1}} + \frac{\partial Y_{piktB}}{\partial RG_{iktB}}\right)(RG_{iktB+1} - RG_{iktB})$$

$$+\frac{\partial Y_{pik}}{\partial PT_{ik}}(PT_{iktB+1}-PT_{iktB})$$

$$+\frac{\partial Y_{pik}}{\partial GDPPC_k}(GDPPC_{ktB+1}-GDPPC_{ktB}) \qquad (13\text{-}43)$$

In those equations, T stands for the time trend where 1961=1, Y_{pik} denotes the potential yield, i is the index of a crop, and k is the country index. Parameters a_{ik}, b_{ik}, c_{ik}, and d_{ik} of functions (13-42) and (13-43) are the same as those in function (13-1). The marginal propensity of potential yield to rainfall is fixed. It is the same as that of function (13-1).

By subtracting function (13-42) from function (13-43), a difference type yield function is derived. The yield function in year t can be written as shown below.

$$Y_{ikt} = Y_{ikt-1} + \frac{b_{ik}-a_{ik}}{1+\exp[-c_{ik}(T_t-d_{ik})]}$$

$$-\frac{b_{ik}-a_{ik}}{1+\exp[-c_{ik}(T_{t-1}-d_{ik})]}$$

$$+\frac{1}{2}\left(\frac{\partial Y_{pikt}}{\partial TP_{ikt}}+\frac{\partial Y_{pikt-1}}{\partial TP_{ikt-1}}\right)(TP_{ikt}-TP_{ikt-1})$$

$$+\frac{1}{2}\left(\frac{\partial Y_{pikt}}{\partial RG_{ikt}}+\frac{\partial Y_{pikt-1}}{\partial RG_{ikt-1}}\right)(RG_{ikt}-RG_{ikt-1})$$

$$+\frac{\partial Y_{pik}}{\partial PT_{ik}}(PT_{ikt}-PT_{ikt-1})$$

$$+\frac{\partial Y_{pik}}{\partial GDPPC_k}(GDPPC_{kt}-GDPPC_{kt-1}) \qquad (13\text{-}44)$$

In a similar fashion, the yield function that is specified as the linear function with the logarithmic time trend is

$$Y_{ikt} = Y_{ikt-1} + b_{Lik}(\ln T_{Lt} - \ln T_{Lt-1})$$

$$+\frac{1}{2}\left(\frac{\partial Y_{pikt}}{\partial TP_{ikt}}+\frac{\partial Y_{pikt-1}}{\partial TP_{ikt-1}}\right)(TP_{ikt}-TP_{ikt-1})$$

$$+\frac{1}{2}\left(\frac{\partial Y_{pikt}}{\partial RG_{ikt}}+\frac{\partial Y_{pikt-1}}{\partial RG_{ikt-1}}\right)(RG_{ikt}-RG_{ikt-1})$$

$$+\frac{\partial Y_{pik}}{\partial PT_{ik}}(PT_{ikt}-PT_{ikt-1})$$

$$+\frac{\partial Y_{pik}}{\partial GDPPC_k}(GDPPC_{kt}-GDPPC_{kt-1}), \qquad (13\text{-}45)$$

where T_L represents the time trend where 1951=1. Parameter b_{Lik} of function (13-45) is the same as that in function (13-2).

The marginal propensities are replaced by elasticities multiplied by the yield by temperature and solar-radiation in the base year tB for estimation in this model as shown below.

$$Y_{ikt} = Y_{ikt-1} + \frac{b_{ik}-a_{ik}}{1+\exp[-c_{ik}(T_t-d_{ik})]}$$

$$-\frac{b_{ik}-a_{ik}}{1+\exp[-c_{ik}(T_{t-1}-d_{ik})]}$$

$$+\frac{1}{2}\left(\frac{\partial \ln Y_{pikt}}{\partial \ln TP_{ikt}}+\frac{\partial \ln Y_{pikt-1}}{\partial \ln TP_{ikt-1}}\right)\frac{Y_{iktB}}{TP_{iktB}}$$

$$\times(TP_{ikt}-TP_{ikt-1})$$

$$+\frac{1}{2}\left(\frac{\partial \ln Y_{pikt}}{\partial \ln RG_{ikt}}+\frac{\partial \ln Y_{pikt-1}}{\partial \ln RG_{ikt-1}}\right)\frac{Y_{iktB}}{RG_{iktB}}$$

$$\times(RG_{ikt}-RG_{ikt-1})+\beta_{PTik}(PT_{ikt}-PT_{ikt-1})$$

$$+\beta_{GDPik}(GDPPC_{kt}-GDPPC_{kt-1}) \qquad (13\text{-}46)$$

$$Y_{ikt} = Y_{ikt-1} + b_{Lik}(\ln T_{Lt} - \ln T_{Lt-1})$$

$$+\frac{1}{2}\left(\frac{\partial \ln Y_{pikt}}{\partial \ln TP_{ikt}}+\frac{\partial \ln Y_{pikt-1}}{\partial \ln TP_{ikt-1}}\right)\frac{Y_{iktB}}{TP_{iktB}}$$

$$\times(TP_{ikt}-TP_{ikt-1})$$

$$+\frac{1}{2}\left(\frac{\partial \ln Y_{pikt}}{\partial \ln RG_{ikt}}+\frac{\partial \ln Y_{pikt-1}}{\partial \ln RG_{ikt-1}}\right)\frac{Y_{iktB}}{RG_{iktB}}$$

$$\times(RG_{ikt}-RG_{ikt-1})+\beta_{PTik}(PT_{ikt}-PT_{ikt-1})$$

$$+\beta_{GDPik}(GDPPC_{kt}-GDPPC_{kt-1}) \qquad (13\text{-}47)$$

In those equations, the β_{PTik} and β_{GDPik} values of functions (13-46) and (13-47) are respectively equivalent to those in functions (13-1) and (13-2).

It is estimated using ordinary least squares (OLS) method or auto regressive (AR) method if the yield function is specified as a linear function. It is estimated using nonlinear least square (NL) method if the yield function is specified as a logistic function.

The temperature and solar-radiation elasticities of yields of the four crops, i.e., *RI*, *WH*, *MZ*, and *SB*, in each country are comparable among different years and countries. Temporary for this model, it is assumed that the temperature and solar-radiation elasticities of yield of other grains, *XG*, and other oil crops, *XS*, are assumed respectively to be equal to those of maize, *MZ* and soybeans, *SB*.

The temperature and solar-radiation elasticities vary according to changes in the climate variables in these yield functions. Therefore, this model is useful for compiling long-run outlooks.

Necessary data for calculation of the potential yields are shown in Tables A-1-1 – A-1-5 in Appendix 1.