Variable density yield model for teak plantations in the Northeast of Thailand

Tosporn Vacharangkura[1)*]

[1)] Forestry Research and Development Bureau, Royal Forest Department, 61 Phaholyothin Rd., Chatuchak, Bangkok 10900, Thailand

* *Corresponding author; e-mail: vtosp@yahoo.com*

Abstract

Variable density yield model for teak plantations in the northeast of Thailand was developed based on 85 temporary sample plots located in the 7 provinces of the northeast *i.e.,* Nakhon Ratchasima, Khon Kaen, Sakon Nakhon, Loei, Si Sa Ket, Ubon Ratchathani and Yasothon. The plots ranged in site index from 14 m to 30 m (dominant tree height at 30 years) and were measured for stand growth 1−4 times during the year 2000−2009. All of measurement plots provided 157 data sets for constructing the model.

The estimation data consisted of 36 growth periods of stands ranging in age from 3 to 40 years. Multiple linear regression method was used to simultaneously fit prediction equations for stem volume, stand basal area, stand volume and stand density. An existing Mitscherlich model for dominant height growth fitted with the same estimation data was used to estimate site quality index of the stands. The models were evaluated both quantitatively and qualitatively. Correlation among error components of the prediction equation for stem volume as well as the equations for stand density, stand basal area and stand volume were strong and statistically significant and the mean biases for those estimators were positive. The presented system of models could serve for constructing variable density yield table of teak plantations that required stand level input data.

Keywords: multiple linear regressions, site index, stand density, stand age

Introduction

Teak (*Tectona grandis* L.f.) is the most important indigenous species of Thailand and is one of the most important tropical hardwood species in the international market of high-quality timber. Teak grows well on variety of geological formations and soils, but best on deep, porous, fertile, well drained sandstones with neutral or acid pH (Kadambi 1972).

Preference of teak in soil is relatively fertile with high calcium, phosphorus, potassium, nitrogen and organic matter contents. According to several studies, teak requires relatively large amounts of calcium for its growth and development (Kaosa-ard 1981). Teak may grow from sea level up to 1,200 meters, but the growth is slower on high elevations and on steep slopes (Kadambi 1972).

In 1994, the Royal Forest Department of Thailand (RFD) launched the Economic Forest Plantation Extension Project to promote forest plantation area. The goal was to cover 800,000 ha (5 million rais) and was designed to encourage rural households to plant trees on their lands. Farmers were granted subsidies of 3,000 baht per rai, over 5 years to plant trees, and were allowed to harvest trees after a certain period. This project emphasized to plant indigenous forest tree species. Teak was the most popular one because of its high durability, good dimensional stability and aesthetic qualities made it a very valuable species for forestry plantations. Additionally, the price of teak wood was relatively high due to the increasing demand. Teak was planted all over Thailand under this project around 88,000 ha during 1994-1996, even in the Northeast where natural distribution of this species does not occur. To reach the end target which provided high valuable timber to the owners of teak plantations, understanding growth and yield were essential for them to develop long-term plan for sustainable forest management. Thus information on growth and yield was high priority to distribute.

A yield table is a table showing the expected timber yields by age of an even-aged stand, usually by site index classes, and typically including quadratic mean diameter (D_q), height, number of stems, basal area, and standing volume per unit area; yield tables may also include volume of thinnings, current annual increment (CAI) (The Dictionary of Forestry 2008). There are three types of yield table *i.e.,* normal yield table, empirical yield table and variable density yield table. Normal yield table is a yield table showing the average development of well-stocked stands over time, usually by site index. Empirical yield table is a yield table usually based on inventory data, showing average volumes and other statistics in relation to age and (sometimes) site index classes as they are found in the existing forest. Variable density yield table is a yield table that includes stand density in addition to site index and age as classification or predictor values.

Many investigators had used multiple linear regression techniques to predict growth and/or yield for total stands or for some merchantable portion of stands (e.g. Bennett et al. 1959; Clutter 1963; Sullivan and Clutter 1972; Murphy 1983; Rinehart and Standiford 1983; Burkhart and Sprintz 1984; Borders and Bailey 1986). These models provided growth and yield estimates for the whole stand as a function of stand level attributes such as age, density and site index as well as interaction among these variables. Stand density, in turn might be taken to be a function of an initial measure of stand density, age and site quality. Site quality, expressed by site index, depended on the dominant height in relation to age (Clutter et al. 1983). Clutter (1963) introduced the notion of compatibility in growth and yield equations by recognizing that the algebraic form of the yield model can be derived by mathematical integration of the growth model. Sullivan and Clutter (1972) extended Clutter's model by simultaneously estimating and cumulative growth as a function of initial stand age, initial basal area, site index and future age. Stand-level variables, such as age, site index, basal area, or number of trees per acre, were used to predict some specified aggregate stand volume. No information on volume distribution by size class was provided; thus resultant equations from this approach were sometimes referred to as whole-stand models (Avery and Burkhart 1994).

In July 1992, a yield prediction table was constructed under Reforestation and Extension Project in the Northeast of Thailand, Phase II (REX II) by RFD-JICA in order to predict yield of teak plantations grown in the Northeast of Thailand and then was revised under RFD-JIRCAS project during 2009-2010 after more data in this region was obtained. This type of yield table was identified to empirical yield table that showed average growth and yield data of the forest stand. Regarding the limitation of empirical yield table that could not always provide reliable data, especially data of the old stand, thus the variable density yield table was considered to obtain more reliable data of growth and yield.

The main objective of this study was to develop a system of equations to predict stand growth and yield of teak stands in the Northeast of Thailand using multiple linear regression model as the estimation procedure which was useful to provide essential information for management of farmer teak plantation in the Northeast of Thailand.

Material and Methods

The data were measured in 85 temporary sample plots. Most of the sample plots were established in 2002 by REX II Project and a few plots were established in 2000 by Assessment of the Potentiality of Re-afforestation Activities in Climate Change Mitigation Project to represent most teak plantation sites in the Northeast of Thailand. The plots were located in the 7 provinces of Nakhon Ratchasima, Khon Kaen, Sakon Nakhon, Loei, Si Sa Ket, Ubon Ratchathani and Yasothon. The plots were measured annually for 2-3 years in order to estimate stand growth. The last measurement was conducted during the year 2007-2008, and each plot was measured twice on an average. At each measurement, tree diameter at 1.3 m height from the ground (DBH), tree height and number of survival trees were recorded. The stem volume of individual tree was computed using the formula developed by Ishibashi et al. (2002):

$$V = 0.000100712\, DBH^{1.89445042}\, H^{0.763796917} \quad (R^2 = 0.98) \quad (1)$$

where V is individual stem volume (m^3), DBH is diameter at 1.3 m height from the ground (cm) and H is tree height (m).

Stand growth parameters (number of trees, average height, average DBH, dominant tree height, stem volume and stand volume) for each plot were calculated. Site index was determined from the dominant tree height and age of each plot, using the dominant tree height growth model developed by Ishibashi et al. (2010). This model used Mitscherlich curve as a guide curve of height-growth:

$$DTH_{gt} = 31.755015621\, [1 - 0.772111113 \exp(-0.027606608\, t)] \quad (R^2 = 0.64) \quad (2)$$

where t is stand age (year) and DTH_{gt} is dominant tree height at age t on the guide curve (m).

The average height of each plot (Hm) could be estimated by the dominant tree height at measurement time (DTH)

$$Hm = 0.976\, DTH - 2.5243 \quad (R^2 = 0.97) \quad (3)$$

(Ishibashi et al. 2010).

1. System of equations

Several basal area and volume prediction models as well as prediction of number of trees (survival trees) were fitted to the data using multiple linear regression model. The best model (equation) was determined by coefficient of determination, residual analysis and biological implication. The simultaneous system of prediction equations consisted of stand basal area, stem volume, stand volume and stand density as shown in the form of multiple linear model:

$$\mathrm{Ln}\, Y = \alpha + \beta_0 \mathrm{Ln} X_1 + \beta_1 \mathrm{Ln} X_2 + \beta_2 \mathrm{Ln}\, X_3 + \ldots + \beta_n \mathrm{Ln}\, X_n \quad (4)$$

where Y is a variable such as stem volume (m^3/tree), stand volume (m^3/ha), stand basal area (m^2/ha), stand density (trees/ha) (survival at measurement time) and average *DBH* (cm), X_1, X_2, X_3…, X_n are stand growth parameters such as stand volume, stand basal area etc., Ln is natural logarithm, α, β_0, β_1, β_2 are unknown parameters to be estimated from the data.

Site index was defined as dominant tree height at the base age. Since the rotation age is often used as the base age, therefore 30 years was adopted as the base age. Since the use of the system of the equation required the estimation of dominant tree height of each plot at measurement time, therefore estimated dominant tree height was computed by the following equation :

$$DTH_t = SI\left(\frac{DTH_{gt}}{DTH_{g30}}\right) \quad (5)$$

where SI is site index value (m), DTH_t is estimated dominant tree height at age t (m), DTH_{gt} is dominant tree height at age t on the guide curve (m) and DTH_{g30} is dominant tree height at age 30 years old on the guide curve.

When DTH_{gt} and DTH_{g30} were substituted in Eq.5, DTH_t could be estimated by Eq. 6: -

$$DTH_t = SI\,\frac{31.755015621\{1-0.772111113\,\exp(-0.027606608\times t)\}}{31.755015621\{1-0.772111113\,\exp(-0.027606608\times 30)\}} \quad (6)$$

Site index curve were produced by Eq. 6 under the range of SI from 14 to 30 (Fig. 1).

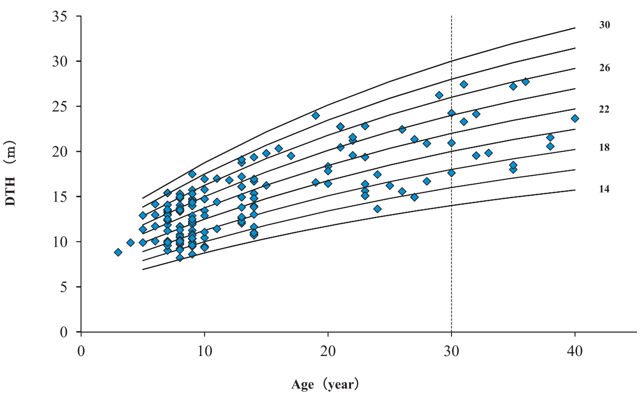

Fig. 1. Site index curve used in this study

2. Model evaluation

The model was evaluated quantitatively and qualitatively using all data for fitting the model. Quantitative evaluation was based on ordinary residuals and the prediction ability of the models was evaluated based on the basis of prediction residuals.

3. Fitting statistics

Quantitative evaluation involved the characterization of model error (bias and precision) and model efficiency (coefficient of determination, R^2). In addition, residuals were examined to detect any obvious pattern and systematic discrepancies. Model bias and precision were evaluated by computing the mean residuals (MRES), the root mean square error (RMSE) and the absolute mean residuals (AMRES) (Eq. 7, 9, and 11). These were also expressed in relative term as percentage of predicted mean value (Eq. 8, 10 and 12).

$$\mathrm{MRES} = \frac{\Sigma\,(y_i - \hat{y}_i)}{\mathrm{n}} \quad (7)$$

$$\mathrm{MRES\%} = 100\frac{\Sigma\,(y_i - \hat{y}_i)\,/\,\mathrm{n}}{\Sigma\hat{y}_i\,/\,\mathrm{n}} \quad (8)$$

$$\mathrm{RMSE} = \sqrt{\frac{\Sigma\,(y_i - \hat{y}_i)^2}{\mathrm{n}-1}} \quad (9)$$

$$\mathrm{RMSE\%} = 100\sqrt{\frac{\Sigma\,(y_i - \hat{y}_i)^2/(\mathrm{n}-1)}{\Sigma\hat{y}_i\,/\,\mathrm{n}}} \quad (10)$$

$$\mathrm{AMRES} = \frac{\Sigma\,|\,y_i - \hat{y}_i\,|}{\mathrm{n}} \quad (11)$$

$$\mathrm{AMRES\%} = 100\frac{\Sigma\,|\,y_i - \hat{y}_i\,|\,/\mathrm{n}}{\Sigma\hat{y}_i\,/\,\mathrm{n}} \quad (12)$$

where n is number of observations and y and $\hat{y}$ are observed and predicted value respectively.

The bias and precision of the prediction models of the system were also examined using different intervals of age class.

Using data sets from 85 independent sample units the goodness-of-fit for all sub-models was also conducted using a bilateral paired t-test. It was used to perform a pair-wise comparison between the observed value and the predicted value computed by the sub-models. The null hypothesis was that there was no significant difference between the actual values and the predicted values. The difference between those values was evaluated to show that whether there is statistically significant difference or not.

The stand volume of different initial-density stands was qualitatively examined by using 45-degrees line test. Since most of teak plantations in Thailand were raised using 4x4m, 2x4m and 2x2m spacing (the initial densities were 625,1250 and 2,500 trees/ha respectively) those three initial stand-densities were selected. The observed values and the predicted values were plotted to examine the trend of the slope of expected curves. If the expected curves tend to make an angle of 45 degrees with the axes, this meant that there was no significant difference between the actual values and the predicted values.

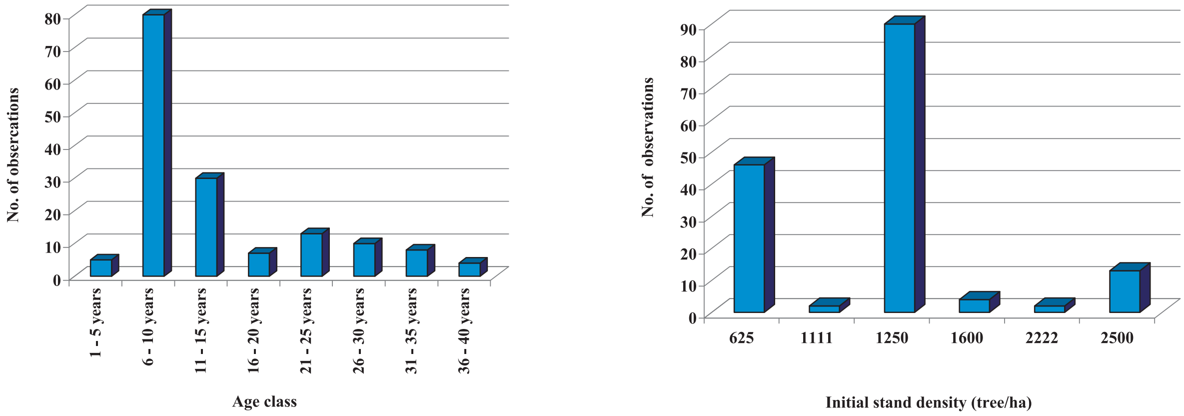

Fig. 2. The characteristics of the 85 temporary sample plots

Table 1. Summary of the characteristics of the 85 temporary plots, as computed from the 157 observations used in the study

Variable	Average (min, max)	SD
Stand age (yr)	14.0 (3, 40)	8.6
Site index (m)	21.0 (14, 29)	3.7
Stand basal area (m^2/ha)	10.9 (3.4, 29.3)	5.3
Stand density (tree/ha)	893.0 (188, 2450)	439.6
Dominant tree height (m)	15.0 (8, 28)	4.4
Stand volume (m^3/ha)	81.0 (17, 268)	54.7
Frequency of measurement per plot	1.9 (1, 4)	1.3

Results and Discussion

The characteristics of 157 observations as derived from the 85 temporary sample plots used in the study shown in Table 1.

The stand characteristics were given for the measurement time of the sample plots. Both of the observations covered various initial densities, however most of them existed on 625 trees/ha (4x4m spacing), 1,250 trees/ha (2x4m spacing) and 2,500 trees/ha (2x2m spacing). Most of stand age ranged from 6-15 years old accounted for 54% of all data sets (Fig. 2). It should be made aware of the restrictions of our data, having a small number of the observations over 30 years old and less than 5 years old of stand age. The number of sample plots which initial stand densities were 1,111 trees/ha and 2,222 trees/ha was very few.

The average DBH of each stand was computed using multiple linear regression model:

$$\mathrm{Ln}\, DBH = 1.3566 - 0.1704\mathrm{Ln}\, I - \mathrm{Ln}\, 1/A + 0.7877\mathrm{Ln}\, Hm \quad (R^2 = 0.96) \qquad (13)$$

where I is initial stand density (trees/ha), $1/A$ is inverse age (1/year), Hm is average height growth (m) and Ln is natural logarithm.

Table 2. Parameter estimates and their standard errors for the sub-models

Dependent variable (Y)	Parameter	Estimate	Standard error
Vt	α	-11.1761	0.5600
(stem volume, m^3/tree)	β_0	-0.4421	0.0580
	β_1	-1.3977	0.0436
	β_2	2.7502	0.0987
V	α	-7.3040	0.6446
(stand volume, m^3/ha)	β_0	0.3123	0.6670
	β_1	0.9543	0.0502
	β_2	2.2898	0.1136
Ba	α	-6.4560	0.6443
(stand basal area, m^2/ha)	B_0	0.3353	0.0667
	β_1	-0.6860	0.0502
	β_2	1.5440	0.1136
N	α	3.8722	6.2073
(stand density, tree/ha)	β_0	0.7544	11.6857
	β_1	0.4434	9.1211
	β_2	-0.4603	-4.1872

Table 3. Cross-equation correlation matrix of residuals of the equations system

	V	N	Vt	Ba
V	1.0000	*0.4976*	*0.3903*	*0.9451*
N		1.0000	*-0.1849*	*0.5903*
Vt			1.0000	*0.2321*
Ba				1.0000

Italic type (significant at $p < 0.05$)

The yield prediction sub-models were derived for stem volume (m^3/tree), stand volume (m^3/ha), or stand basal area (m^2/ha) or stand density (trees/ha). Various parameters of stand growth were chosen to fit the sub-models. It was found that the appropriate independent variables of the sub-model were the natural logarithm of initial stand density,

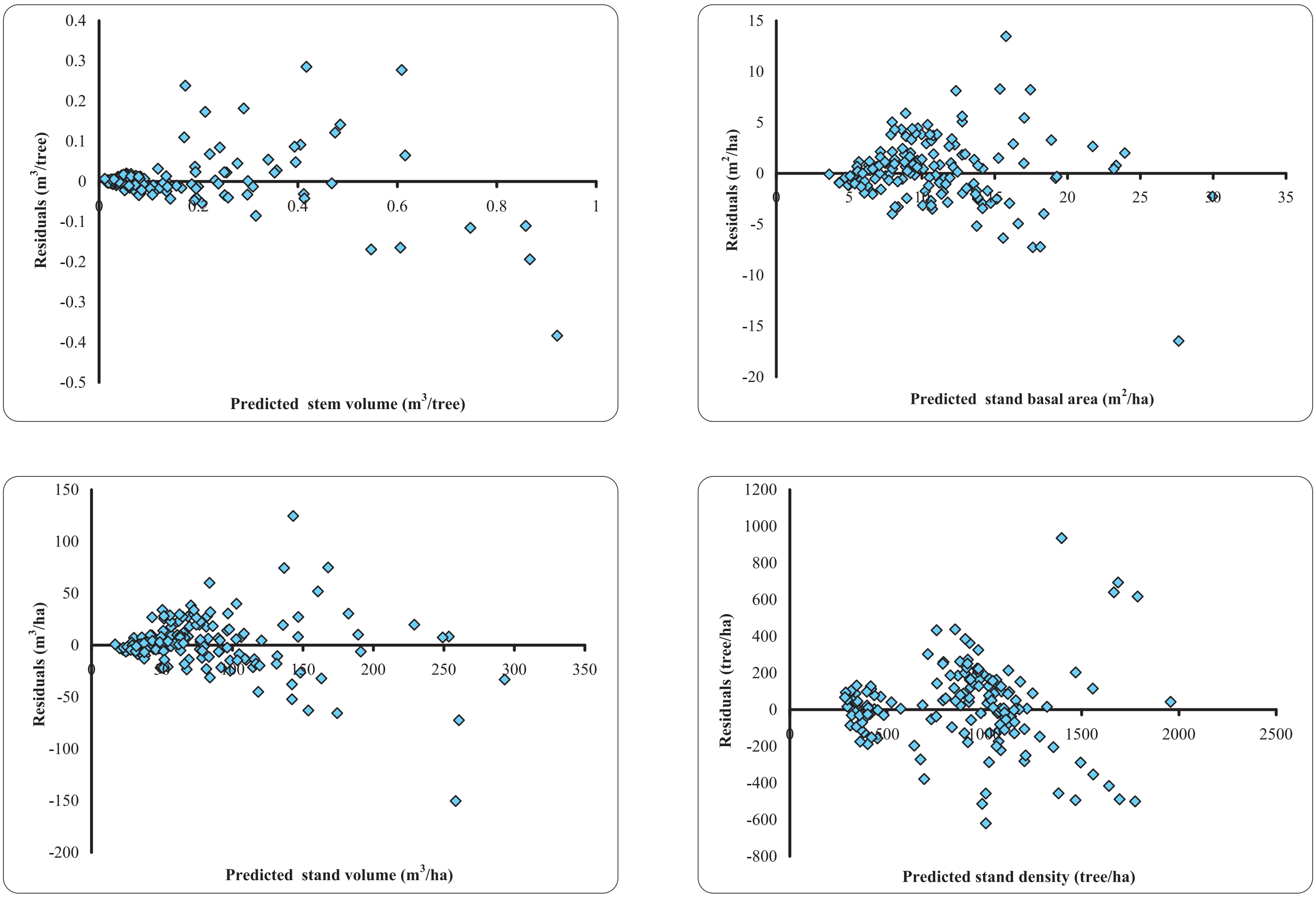

Fig. 3. Residual versus predicted values for the sub-models of stem volume, stand volume, stand basal area and stand density

Table 4. Characterization of error for Multiple Linear Regression Equations

	R^2	MRES	MRES%	AMRES	AMRES%	RMSE	RMSE%
V	0.85	1.6736	2.10	16.7799	21.05	2.1388	2.68
Vt	0.96	0.0030	2.05	0.0303	20.59	0.0053	3.57
Ba	0.72	0.2918	2.76	2.1694	20.51	0.2549	2.41
N	0.81	22.2881	2.56	151.0032	17.33	2.0170	2.02

inverse age and site index value and the dependent variables were the natural logarithm of the above stand aggregates:

$$\text{Ln } Y = \alpha + \beta_0 \text{Ln} I + \beta_1 \text{Ln} 1/A + \beta_2 \text{Ln} SI \quad (14)$$

where Y is a variable, such as stem volume (m^3/tree), stand volume (m^3/ha), stand basal area (m^2/ha) and stand density (trees/ha) (survival at measurement time), I is initial stand density (trees/ha), $1/A$ is inverse age (1/years), SI is site index value (m) and Ln is natural logarithm.

The parameters estimates of the sub-models (Eq. 14) as well as their associated standard error were shown in Table 2. The coefficient of determination (R^2) of all sub-models was quite high. All parameters estimates were logical and significant at 0.01 level. Correlation among error components for stand volume, stand density, stem volume and stand basal area were significant at 0.05 level (Table 3). A negative cross-equation correlation of residuals between the stem volume and stand density at the same age means that if the stem volume is overpredicted, it is likely that the stand density is underpredicted.

This study presented stand-level growth and yield models for teak plantations in the Northeast of Thailand. The sub-models for stem volume, stand volume, stand basal area and stand density were fitted simultaneously using multiple linear regression, while the dominant tree height growth model developed by Ishibashi et al. (2010) was used independently to predict dominant tree height of teak stand.

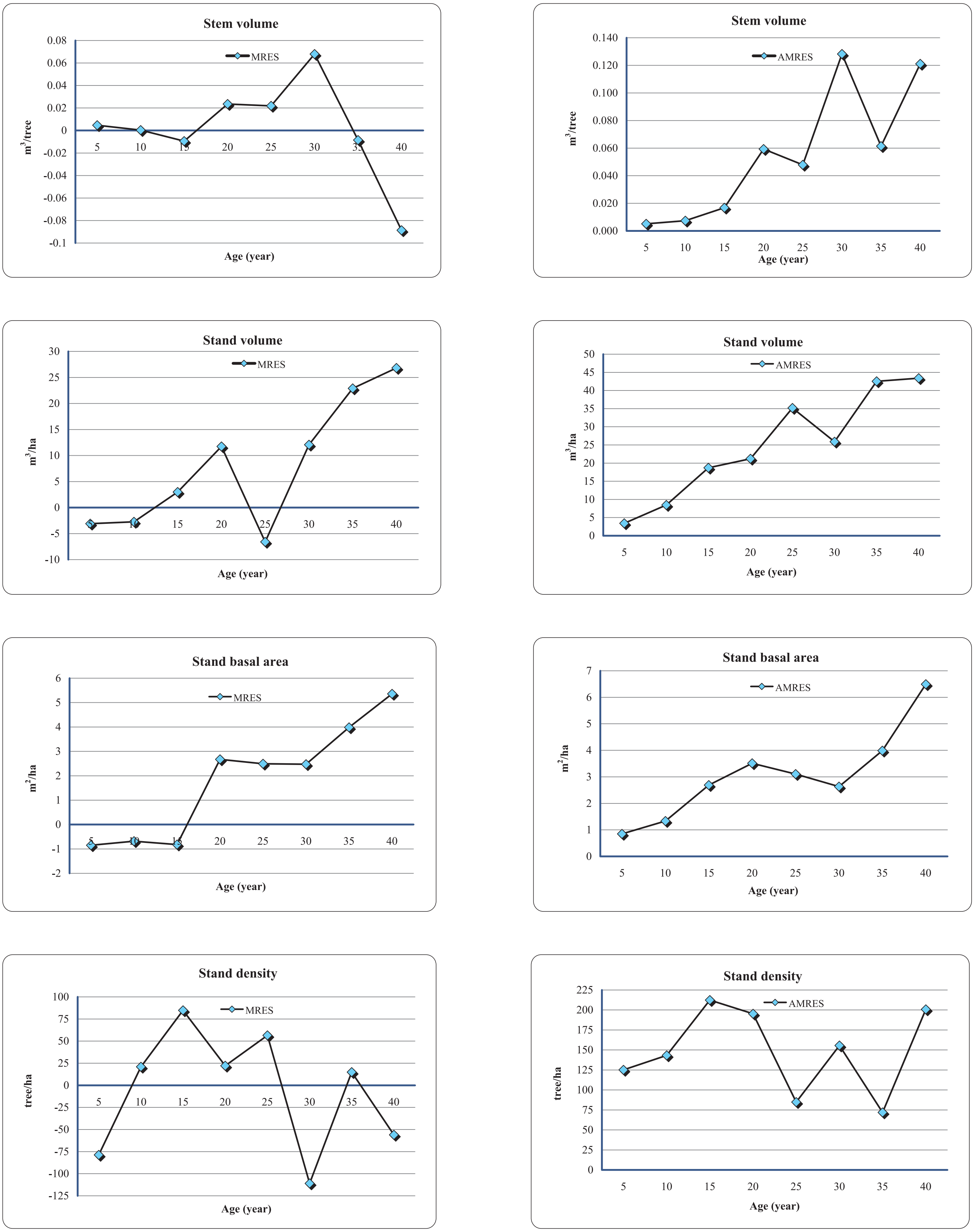

Fig. 4. MRES (model bias) and AMRES (model precision) for the prediction sub-models of stem volume, stand volume, stand basal area and stand density

Table 5. The results of paired sample *t* - test of the yield prediction

	t - value	*p* - value
V	0.7865	0.4327^{ns}
Vt	0.5759	0.5655^{ns}
Ba	1.1531	0.2506^{ns}
N	1.2792	0.2027^{ns}

Significant level ($p < 0.05$), ns : not statistically significant.

Since the dominant tree height derived the system of the yield model and it was also used to estimate the site index of a given stand, it had to be assured that a robust and reliable prediction model for dominant tree height was available.

There were no serious patterns on the distribution of residuals in the stem volume, stand basal area, stand volume and stand density (Fig. 3).

MRESs were positive for all multiple linear regression equations (sub-models) of the stem volume, stand basal area, stand volume and stand density (Table 4), showing that some positive bias exists in the model. MRES%s and MRESs as well as RMSE%s and RMSEs which measured bias, were quite small. This meant that the sub-models provided accurate prediction. On the contrary, AMRES%s and AMRESs which measured precision, were rather large around 17%-20% for all sub-models.

Fig. 4 showed the bias (MRES) and precision (AMRES) by each of the prediction sub-models of the system using different predictions intervals. The predictive ability of all sub-models decreased as the stand age increased.

The bias in stem volume, stand volume and stand basal area from 5 years to 15 years of interval were smaller than the longer interval which meant that the sub-models gave more accuracy for 5-15 year-old stands than the older stand. The stand density sub-model provided accurate prediction for 10, 20 and 35 years of interval.

The AMRESs in stem volume, stand volume and stand basal area from 5 years to 15 years of interval were smaller than those of longer interval which meant that the sub-models gave more precision for 5-15 year-old stands than the older stand. The stand density sub-model gave more precision for 5, 10, 25 and 35 years of interval than other intervals of ages. It could be concluded that the stem volume, stand volume and stand basal area sub-models gave more prediction ability for 5-15 year-old stands than the older stand.

The predicted stand volume was underestimated when the stand was approximately 5, 10 and 25 years old, while it gave overestimate when the stand was approximately 15, 20, 30, 35 and 40 years. The prediction ability in the stand volume showed smallest when the stand was 40 years old.

For a more statistically comparison in the goodness-of-fit of all sub-models, the observed values of all sample plots were collectively compared with the corresponding values

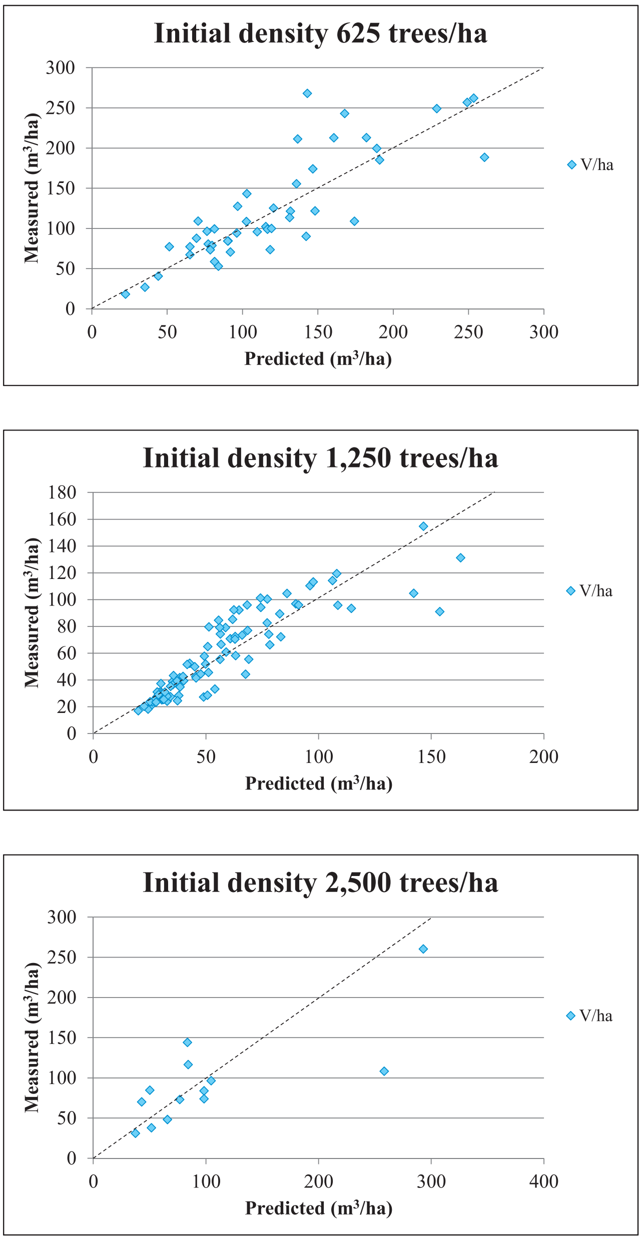

Fig. 5. Comparison of the 45 degree line test

predicted by the yield prediction equations. The comparisons were made with the help of paired sample t-test. These implied that the observed values of all predictions (stem volume, stand volume, stand basal area and stand density) were not significantly different from those predicted values at 0.05 level (Table 5). Thus the system of yield prediction model was acceptable.

Using graphical method, the model was also qualitatively evaluated by comparison of stand volume among the various initial stand densities. Fig. 5 showed the relationship between actual (measured) value and predicted value of stand volume. The trend of the slope of expected curves compared among different initial stand densities (625, 1,250 and 2,500 trees/ha) was less different from 45 degrees line. It can be observed that the models tended to make an angle of 45 degrees with the axes, meaning that there was no significant difference between the actual yield and the predicted yield. The model tended to give both overestimation and underestimation of the actual yields for each initial-density stand.

The construction of variable density yield table was based on the set of models developed in this study and on the dominant tree height growth model developed by Ishibashi et. al. (2010). The yield table was constructed through the following steps:

1. For each stand, compute the dominant tree height by using Eq. 2 and compute the site index value by using Eq. 6.
2. Compute the average tree height from the dominant tree height by using Eq. 3.
3. Predict the average diameter of the stand at the prediction age by using Eq.13.
4. Predict stem volume, stand volume, stand basal area (Eq.14) based on initial stand density, inverse stand age (stand age at prediction time) and site index value.

Conclusion

The variable density yield model developed in this study could be implied to construct the yield prediction table for teak plantation in the Northeast of Thailand after the confirmation of the sub-models through the validity test. Although the developed model showed a high reliability of prediction ability, it had certain limitations for the system of equations used in this model. It should only be used for predicting growth and yield of teak stand in the Northeast of Thailand where all of data were collected. The application of the model must be in the range of these data. In addition, the use of the model was also based on age limitations. The results from validity test confirmed that the accuracy and precision of the model were best for the stand age ranged from 5-15 year-old. If the stand age was beyond this range, the predicted values were likely to have more bias and inconsistency.

Acknowledgments

The author thanks the staffs of Forest Management Bureau in Nakhon Ratchasima, Udon Thani and Yasothon for their valuable assistances. Financial support for this study was give by the Royal Forest Department and JIRCAS. We gratefully acknowledge the data provided by the Reforestation Extension in Northeast of Thailand Project (REX Project) and the collaborative project between the Royal Forest Department and Japan International Forestry Promotion Organization (JIFPRO).

Reference

Avery TE, Burkhart HE (1994) Forest Measurement. 4th ed. Mc Graw-Hill Book Co., New York

Bennett FA, McGEE CE Clutter JL (1959) Yield of old-field slash pine plantations. USDA For. Serv. Stn. Pap.107:1-19

Borders BE, Bailey RL (1986) A compatible system of growth and yield equations for slash pine fitted with restricted three-stage least squares. For. Sci. 32:185-201

Burkhart HE, Spring PT (1984) Compatible cubic volume and basal area projections for thinned old-field Loblloly pine plantations For. Sci. 30:86-93

Clutter JL (1963) Compatible growth and yield models for Loblolly pine. For. Sci. 9: 354-371

Clutter JL, Forston JC, Pienaar LV, Brister GH Bailey RL (1983) Timber management-a quantitative approach. John Wiley & Sons Inc., New York

Ishibashi S, Ishida H, Okabayashi M, Sirilak S, Wanussakul R, Klaikaew N, Intuman C, Cha-umpol C (2002) Yield Prediction Table on *Tectona grandis* (Teak) in Northeast Thailand.(Study Report) 11th July 2002. The Reforestation and Extension in the Northeast of Thailand Phase II(REX II).20pp

Ishibashi S, Sakai M, Noda I, Vacharangkura T, Krongkitsiri V, Kamolpanit D, Himmapan W (2010) Yield Prediction Table on *Tectona grandis* (Teak) in Northeast Thailand (Revised edition). RFD-JIRCAS Joint Research Project, Bangkok

Kadambi K (1972) Silviculture and Management of Teak. Bulletin, No. 24, 137 pp.,71 ref., School of Forestry Stephen F., Austin State University

Kaosa-ard A (1981) Teak : Its natural distribution and related factors. Nat. His. Bull. Siam. Soc. 29: 55-74

Murphy PA (1983) A nonlinear timber yield equation system for Loblolly pine. For. Sci. 29: 582-591

Rinehart James A, Richard B, Standiford (1983) Growth and Yield in *Eucalyptus globulus.* Gen. Tech. Rep. PSW 69, Berkeley, CA: Pacific Southwest and Range Station, Forest Service, U.S. Department of Agriculture

Sullivan AD, Clutter JL (1972) A simultaneous growth and yield model for Loblolly pine. For. Sci. 18:76-86

The Dictionary of Forestry (2008) Yield Table. Source: http: //dictionaryofforestry.org/dict/term/yield_table, Society of American Forestry, Copyright 2008, May 6, 2011