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Taper Equations for Evaluating Private Plantation Teak (*Tectona grandis*) in Thailand

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Abstract
We applied data from 407 trees (5-33 years old) from private teak (*Tectona grandis*) plantations to three typical taper models, including the Goodwin cubic polynomial model comprising hyperbolic and parabolic terms and the Kozak variable-exponent taper model. On the basis of the three models, 18 variants were fitted using nonlinear regression analysis. All models were defined to predict stem diameters overbark using diameter at breast height overbark. A bark thickness equation was prepared to convert overbark diameters to ones excluding the bark. Goodness-of-fit and leave-one-out cross-validation appraisals were used to select the best model. A variant of the Kozak model (Model K8) performed the best across three prediction tests: diameter given height, height given diameter, and log volume given two heights. Taper equation K8, derived from Model K8, provided actual values within the 10% mean error and was sufficiently accurate and precise at the valuable bole part. Teak trees in our study were different in stem form and slender (a high value in slenderness coefficient) compared to those in the state-owned Forest Industry Organization (FIO) teak plantations, and the use of the FIO taper model for slender stems was challenging. Trees in the private plantations generally had thicker barks than those in the FIO plantations. We concluded that equation K8 is recommended for private teak plantations in this study area. These results will contribute to studies on teak taper equations and bark thickness in Thailand.

Discipline: Forestry
Additional key words: bark thickness, cross-validation, slenderness coefficient, stem form, sustainable forestry

Introduction

Teak (*Tectona grandis*) is grown in smallholder agroforestry systems in many tropical countries (Roshetko et al. 2013). Through the Economic Tree Planting Extension Project, a Thai government subsidy program aimed at combating deforestation, farmers established 351,000 ha of tree plantations, including 151,000 ha of teak plantations from 1994 to 2001 (RFD 2002). However, most teak plantation farmers require suitable techniques to readily estimate the fair merchantable value of a standing tree (Furuya et al. 2012a, Himmapan et al. 2010, Noda & Himmapan 2014, Pusudsavang et al. 2012).

Because the price of a teak log is determined primarily by its diameter (Keogh 2008, Noda et al. 2012, Wijenayake et al. 2019), the use of taper equations is essential to obtain both log and stem volume and diameter at any height above ground and estimate the fair merchantable value or maximize the log product value of a standing teak tree (Warner et al. 2016). As Kozak (2004) summarized, although volume equations only predict the total or merchantable stem volume, taper equations predict, among others, the diameter at any height, merchantable height to any top diameter, log volume of any length at any height above ground, and merchantable

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volume. Thus, taper equations are fundamental to many forest harvesting systems, where they are used primarily in log bucking systems (Goodwin 2009). It is worth noting that tree stem forms can be affected by silvicultural practices because stem form or tree slenderness is affected by environmental factors, including stand density (Newberry & Burkhart 1986, Wang et al. 1998, Watt & Kirschbaum 2011, Zeide & Vanderschaaf 2002).

Taper equations have been used for more than a century by foresters. Many models of varying degrees of complexity have been described by numerous authors because of the difficulty in formulating generalizable equation components for even a single species (Van Zyl 2005). However, taper models are classified into the following: 1) variable-exponent models using a changing exponent to describe the neiloid, paraboloid, and conical forms of the stem (Kozak 1988, Perez et al. 1990); 2) polynomial models, including a cubic polynomial (Goodwin 2009, Osumi 1959, Yamada & Ohno 2016) and a polynomial with powers of the Fibonacci sequence numbers (Laasasenaho 1982); and 3) segmented models using a different submodel for the partitioned domain to overcome various biases (Max & Burkhart 1976). However, the application of segmented models can be complicated (Van Zyl 2005).

Taper equations have been used in assessing various characteristics of teak plantations in some countries (Van Zyl 2005). In Thailand, two taper equations for teak plantations have been reportedly used: "FIO-teak1" in northern regions and "TPP2" in western regions developed by Warner et al. (2016) and Choochuen et al. (2021), respectively. Warner et al. (2016) evaluated the variants of the polynomial model (Goodwin 2009) and the variable-exponent model (Kozak 2004). They found that a suitable taper model for teak plantations in northern Thailand is a Goodwin model variant (called Goodwin-X3A), using which they established a taper equation termed "FIO-teak1." Choochuen et al. (2021) suggested that for teak plantations in western Thailand, the best taper model among Goodwin model variants, was the same as that which outputted "FIO-teak1," and the taper model/equation was named "TPP2." However, these two studies focused only on teak plantations managed by the state-owned Forest Industry Organization (FIO) and not on privately managed ones, the latter being the focus of our study.

The FIO manages teak plantations under the following rules: a planting distance of 4 m × 4 m; regulated thinning at 10, 15, and 20 years of age; and a rotation length of up to 30 years (Himmapan & Noda 2012, Korwanich 1992). Additionally, Choochuen et al. (2021) noted that the teak trees in plantations managed by the FIO in western and northern Thailand have similar stem forms because the seedlings are likely clones with the same genetic basis. In contrast, Thai private teak planters use seedlings with various genetic bases; select narrow planting distances, such as 2 m × 4 m (i.e., 1,250 trees/ha) or 2 m × 2 m due to requirements of the subsidy program (Furuya et al. 2012b); and do not conduct regular thinning because of unfamiliarity with the technique (Furuya et al. 2012a). Thus, we hypothesized that an equation different from the available taper equations is required for private teak trees because of their slender stem forms caused by denser stand conditions. We could not find any studies on taper equations for private teak plantation trees in Thailand, including the application of FIO-teak1 or TPP2 to private teak plantation trees.

The objectives of our study were the following: 1) to select several models producing fitting taper equations from various taper models, including the Goodwin-X3A model, using available data from privately managed teak plantations in Thailand, 2) to evaluate the performance differences of some aspects of the selected models to determine the best taper equation, and 3) to establish the effectiveness of the taper equation determined in step 2 relative to that of FIO-teak1 in the context of private teak plantation trees.

Materials and methods

1. Data collection

This study used data from 407 sample trees in private teak plantations throughout northern, northeastern, central, and western Thailand (Table 1 and Fig. 1). The area belongs to a dry tropical savanna (Aw) type with annual rainfall in the range of 1,000 to 1,500 mm. Sample trees were selected from dominant and regularly formed (lacking damages) trees. This study included private teak plantations with a planting distance narrower than that of the FIO standard (4 m × 4 m) to focus on current conditions (Table 1). We felled 103 of the 407 trees for sectional measurements. The diameter overbark (d_{ob}) was measured with a diameter tape or caliper at heights of 0.3 and 1.3 m and then at consecutive 1-m intervals until the top. Bark thickness was measured with a bark gauge (Haglöf Inc., Långsele, Sweden) at four orthogonal directions to calculate its mean value at the position of d_{ob} measurement after tree height (H) was measured with a measuring tape. Additionally, without felling, H and d_{ob} of 304 sample trees were measured using a Vertex IV ultrasound distance measurer (Haglöf Inc.) and a Criterion RD1000 electronic dendrometer (Laser Technology Inc., Centennial, CO, USA),

Table 1. Summary of the 407 teak sample trees by province

Province	No. of trees	Age (y)	DBH (cm)	Tree height (m)	Planting distance*[1]
Chiang Mai (N)	10	11	12.2-16.8 (14.6±1.5)	12.1-16.2 (14.4±1.1)	2 m × 4 m
Kanchanaburi (W)	5	20	7.0-23.0 (14.8±6.5)	9.2-17.6 (14.7±3.3)	2 m × 4 m
Kham Phang Phet (N)	10	22	8.3-24.2 (15.5±5.5)	12.4-19.8 (16.3±3.0)	2 m × 3 m
Loei (NE)	90	15-23 (20.7±2.6)	13.3-36.9 (20.9±4.8)	12.8-27.4 (20.0±3.0)	2 m × 2 m (30) & others*[2]
Lopburi (C)	15	9-11 (10.3±1.0)	12.0-27.0 (16.7±4.6)	12.2-20.3 (15.4±2.3)	3 m × 3 m
Nongbua Lamphu (NE)	12	15-21 (17.0±3.0)	5.7-24.4 (12.7±5.3)	6.3-20.1 (12.6±4.6)	2 m × 3 m (8) & 2 m × 4 m (4)
Uttaradit (N)	265	5-33 (13.6±4.8)	6.6-32.8 (19.2±3.9)	8.1-21.7 (17.0±2.2)	2 m × 4 m (137) & others*[3]
Total	407	5-33	5.7-36.9	6.3-27.4	
Mean±SD		(15.4±5.3)	(19.0±4.6)	(17.4±3.0)	

Alphabets in parentheses indicate region name: N = northern, NE = northeastern, C = central, W = western region.
DBH: diameter at breast height overbark
*[1]: Numbers in parentheses indicate number of trees by planting distance.
*[2]: 2 m × 2 m (30), 2 m × 4 m (21), 2 m × 3 m (20), 3 m × 3 m (10), and 3 m × 6 m (9)
*[3]: 2 m × 4 m (137), 4 m × 4 m (119), and 2 m × 3 m (9)

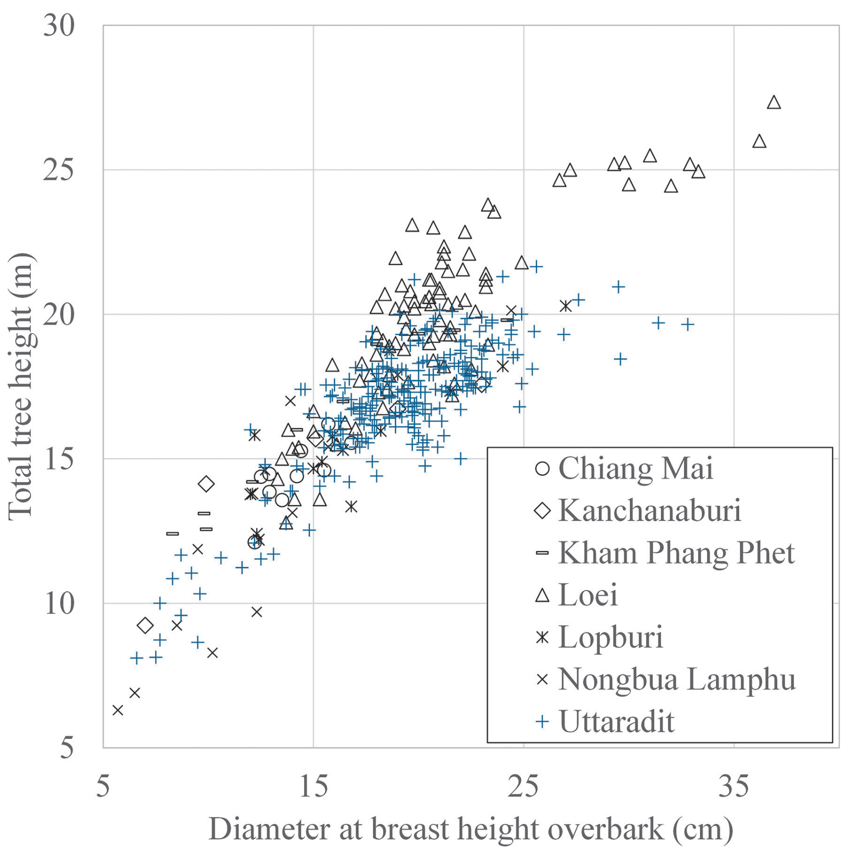

Fig. 1. Tree height and diameter at breast height overbark by plantation (n = 407 trees)

respectively. The d_{ob} was calculated as the average of two orthogonal direction measurements at heights of 0.3 and 1.3 m, followed by consecutive 1-m intervals until the maximum possible height allowed by RD1000. Nondestructive measurement of upper diameters using RD1000 can be considered valid and useful for developing taper equations and tree volume estimation (Marchi et al. 2020, Rodriguez et al. 2014).

2. Determination of models

Variables for taper equations are based on available data. Information on stem diameter underbark (d_{ub}) data is often practically desirable than d_{ob} data because of the limited value of the bark (Keogh 2008, Li & Weiskittel 2011, McTague & Weiskittel 2021, Noda & Himmapan 2014). However, obtaining d_{ub} measurements is usually expensive and time-consuming when conducted on standing trees (Li & Weiskittel 2011). The FIO-teak1 and TPP2 equations require diameter at breast height underbark (DBH_{ub}), which is difficult to measure in the field. Therefore, DBH_{ub} is generally estimated from the diameter at breast height overbark (DBH_{ob}) data using a formula that integrates bark thickness calculations so that DBH_{ub} as input values can have an estimation error. Moreover, the equations of FIO-teak1 and TPP2 with d_{ub} as an objective variable are not easy to assess and improve because of difficulty in data acquisition. Therefore, to develop a suitable taper equation for private teak plantation trees in Thailand, we defined taper models for d_{ob} prediction using DBH_{ob} rather than DBH_{ub} and prepared a bark thickness equation to obtain d_{ub} from the predicted d_{ob}.

(1) Taper models

We selected three taper models, namely, Eqs. 1-3, which have been used or recommended in former studies for teak plantations globally. Variants of those models were appraised by removing terms to test their applicability to teak trees in this study. The following definitions for variables were used in Eqs. 1-3: h = height above ground (m), d = diameter overbark (cm) at height h, H = tree height (m), h_1 = breast height (m) (h_1 = 1.3 m), and D = diameter overbark (cm) at breast height.

Goodwin's model is a cubic polynomial model comprising hyperbolic and parabolic terms. It has been applied to a range of species in Australia, including teak, which was possible because of the provision of a useful super-set of second-stage models (β_1, β_2, and β_3) before the elimination of terms (Goodwin 2009). Warner et al. (2016) used the Goodwin model shown as Eq. 1 to appraise the suitability of the Goodwin model and concluded that a variant model of the Goodwin model was the best for FIO-managed teak plantations in northern Thailand compared to the Kozak model described as Eq. 2.

$$d = (H - h)[S + \beta_3(h - h_1) + D/(H - h_1)], \quad (1)$$

where $S = \beta_1 \beta_2^{\,2}(h_1 - h) \, / \, [(1 + \beta_2 h)(1 + \beta_2 h_1)(1 + \beta_2 H)]$

$$\beta_1 = a_0 + a_1 H + a_2 H^2 + a_3 (D/10)^2$$

$$\beta_2 = b_0 + b_1 H + b_2/H$$

$$\beta_3 = c_0 + c_1 H + c_2/H + c_3(D/10) + c_4(D/10)^2$$

and a_0 to a_3, b_0 to b_2, and c_0 to c_4 are second-stage candidate coefficients.

The Kozak variable-exponent taper model (Eq. 2) was selected for use in this study as it was found to be the best taper model among various Kozak models. It has been applied to the study of many species globally (Kozak 2004, Warner et al. 2016).

$$d = a_0 D^{a_1} H^{a_2} X^M, \quad (2)$$

where $M = b_1 (h/H)^4 + b_2 \left(1/e^{(D/H)}\right) + b_3 X^{0.1} + b_4(1/D) + b_5 H^Q + b_6 X$

$$X = [1 - (h/H)^{1/3}]/[1 - (1.3/H)^{1/3}]$$

$$Q = 1 - (h/H)^{1/3}$$

and a_0 to a_2 and b_1 to b_6 are coefficients.

Equation 3 consists of the ratio of a response variable of diameter to a basic diameter. The basic diameter should be at the lowest height along the stem so that the diameter is not affected by buttress swelling (Osumi 1959). Many studies have used the basic diameter of $d_{0.9}$ (i.e., diameter at length 0.9 from tree top to ground) (Osumi 1959, Wijenayake et al. 2019, Yamada & Ohno 2016). Thus, $d_{0.9}$ was used as the basic diameter in this study.

Equation 3 is a variable-form cubic polynomial model used in a taper model of *Larix kaempferi* in Japan presented by Yamada & Ohno (2016). The Yamada taper model has second-stage models (α_1, α_2, and α_3) of linear functions with tree height and diameter at breast height to correspond to various stem shapes dynamically. A simple cubic polynomial taper model without the second-stage models in Eq. 3 was used for teak plantations in Sri Lanka by Wijenayake et al. (2019). However, studies using the Yamada model to establish a teak taper model were not found. Therefore, we added the Yamada model to our model assessment.

$$d/d_r = \alpha_1 L^3 + \alpha_2 L^2 + \alpha_3 L, \quad (3)$$

where $L = 1 - (h/H)$

$$\alpha_1 = a_1 + a_2H + a_3D$$
$$\alpha_2 = b_1 + b_2H + b_3D$$
$$\alpha_3 = c_1 + c_2H + c_3D$$

and L is the relative length from the tree top to height h (m) above ground, r is the relative length from the tree top to the basic diameter height above ground, d_r is the diameter overbark (cm) at position r (i.e., basic diameter), and a_1 to a_3, b_1 to b_3, and c_1 to c_3 are second-stage candidate coefficients.

For the prediction of d in Eq. 3, first, d_r was calculated with linear interpolation using measured stem diameters because the basic diameter of an object tree was not measured. Second, the coefficients of Eq. 3 were estimated using nonlinear regression of Eq. 4.

$$d = d_r\varphi(L), \tag{4}$$

where $L = 1 - (h/H)$

and $\varphi(L)$ is the right-hand side of Eq. 3.

Appraisals of each model were started with full coefficients, and insignificant terms ($P \geq 0.05$) were removed in stages insofar as there were comparatively big gaps in t-values. Each model was named as the first letter of the model author's surname and the number of coefficients.

(2) Bark thickness model

To obtain a bark thickness equation based on our field data, we used the same power model (Eq. 5) as that of Warner et al. (2016) and Choochuen et al. (2021) in Thailand.

$$BT2 = a(d_{\mathrm{ob}})^b, \tag{5}$$

where $BT2$ is the doubled bark thickness (cm) for d_{ob} (cm).

3. Evaluation of taper models

Taper equations were produced by fitting the 18 taper models using nonlinear regression (nls function in R). Goodness-of-fit (GOF) statistics were calculated to select better performing models for further analysis based on the 5,183 diameter samples collected from 407 trees. Among the 18 models tested, the model named as Goodwin-X3A by Warner et al. (2016) was included as Model G5x. The adjusted coefficient of determination ($R^{2\prime}$), the residual standards error, and the Bayesian information criterion were used for GOF appraisal because these have often been used in comparisons of prediction models (Ritz & Streibig 2008, Shmueli 2010, Warner et al. 2016).

Better models in the GOF analysis were evaluated by leave-one-out (LOO) cross-validation (Maindonald & Braun 2003) to clarify differences in the responses on the following tests. LOO cross-validation was applied to each of the 407 trees to produce estimates for the excluded tree based on the model fit using the remaining 406 trees. Percentage error (PE%) and the relative root mean square error (RRMSE%) were used as the cross-validation statistics to provide averaged measures of the overall model performance (Huang et al. 2003, Warner et al. 2016). PE% deals with accuracy to indicate the size and signs (negative value, overestimation; positive value, underestimation) of the prediction errors (Eq. 6), and RRMSE% portrays the precision of the model prediction, i.e., the root mean square error on a relative scale, relative to mean value of measurements (Eq. 7) (Huang et al. 2003). The range of precision of the models in this study was considered excellent, <10%; good, 10%-20%; fair, ≥20%-30%; and poor, ≥30% based on RRMSE% (Jamieson et al. 1991, Li et al. 2013).

$$PE\% = 100\left[\textstyle\sum_i^n((y_i - \hat{y}_i)/n)\right] / \bar{y}, \tag{6}$$

$$RRMSE\% = 100\left[\sqrt{\textstyle\sum_i^n((y_i - \hat{y}_i)^2/n)}\right] / \bar{y}, \tag{7}$$

where n is the number of data, y_i is a measured value, $\hat{y}_i$ is its predicted value, and $\bar{y}$ is the mean of y_i. We used a mean prediction error (estimated/actual) of 10% for the model acceptability threshold to assess the suitability of a model (Choochuen et al. 2021, Huang et al. 2003, Warner et al. 2016).

LOO cross-validation tested a model on three aspects: 1) prediction of d_{ob} given h, 2) prediction of h given d_{ob}, and 3) prediction of log volume overbark given two heights selected at random (v given hh), as per the process followed by Warner et al. (2016). For diameter and height predictions (tests 1 and 2), LOO statistics were calculated within 10 equal-width classes of relative height (RH, the ratio of h to H). For volume estimates (test 3), five equally populated classes of sorted DBH_{ob} were used. An index value of each test was used as the mean of test statistics scaled in a way that each statistic was divided by the test mean across all models. The scaling gave each statistic an equal contribution to the index in each class, and perfect models have an index of zero (Goodwin 2009). Finally, the three test indices were averaged as an overall statistic to determine the best model.

In the LOO procedure, the records where h = 1.3 were omitted because the residuals were already constrained to zero by the Goodwin model. Furthermore, to reduce the potential correlation between measurements in the same tree, only one value selected randomly from each tree in each subclass of the tree stem was used in each LOO procedure (Warner et al. 2016). An actual value of log volume was calculated using Smalian's formula (Avery & Burkhart 2002).

4. Comparison to the FIO-teak1 equation

To clarify the effectiveness of our new taper equation, we used the taper equation derived from the best taper model and taper equation G5x derived from Model G5x having the same formation as the FIO-teak1 equation. Results of the prediction test of stem diameter given height were compared between our taper equations and the FIO-teak1 equation. The bark thickness equation was used to obtain DBH_{ub} from DBH_{ob} and d_{ub} from d_{ob} predicted using our taper equations of the best model and Model G5x for the input of FIO-teak1. The taper equations' performances were evaluated by calculating PE% and RRMSE% in each RH class by applying to the dataset (n = 407 trees).

Furthermore, responses to the different slenderness of trees were compared among the taper equations to investigate differences in stem forms between our data and FIO plantations because denser stands tend to foster slender trees (Wang et al. 1998, Zeide & Vanderschaaf 2002). The taper equations were applied to subsets classified by the magnitude of the slenderness coefficient (SC, i.e., the ratio of H to DBH) (Wang et al. 1998).

We conducted all analyses using R v4.0 (R Core Team 2021) and referred to sample R scripts used by Warner (2016). The stratified R function made by Ananda Mahto was used to randomly select one record for each unique value in a specified column for the LOO analysis, as shown in Warner (2016). The uniroot and integrate functions were used to predict h given d_{ob} and estimate log volume, respectively.

Results

1. Evaluation of taper models

Standard residuals of the 18 model fittings showed no indications of serious bias or trends under the GOF analysis (Fig. 2). As a result, seven models (four Goodwin and three Kozak models) were selected for LOO assessment based on their relatively better performance (i.e., ΔBIC = 0 to 127) (Table 2). The Kozak models showed better fitting than all Goodwin models. Model G5x was less fitting than the seven selected models. The Yamada models could not provide a good fit for diameter prediction compared to the other models. Model Y3 ($R^{2\prime}$ = 0.95895) had the same formula as that used in Sri Lanka by Wijenayake et al. (2019) but was the worst among the 18 models for GOF appraisal (Table 2). Model Y9 performed the best among the Yamada models but was not selected as one of the seven models because of its relatively worse fitting than that of the Goodwin models.

The index values of the LOO cross-validation indicate that the Kozak models (K7, K8, and K9) were better than all of the Goodwin models based on three prediction tests and that Model K8 was the best among all models across all tests in our study (Table 3). The histograms of the prediction error (estimated/actual) based on the LOO cross-validation generally displayed a normal distribution (Fig. 3).

The LOO cross-validation results of Model K8 are summarized in Table 4. The predicted d_{ob} given h displayed sufficient accuracy (PE% within ±1%) and excellent precision for RH ≤ 70%. That is, the mean error by the RH class was <1.2% of the actual value in the main bole part (RH ≤ 70%), including the valuable bole part, and most d_{ob} values were predicted within 10% error of the actual value. Similarly, the predicted v given

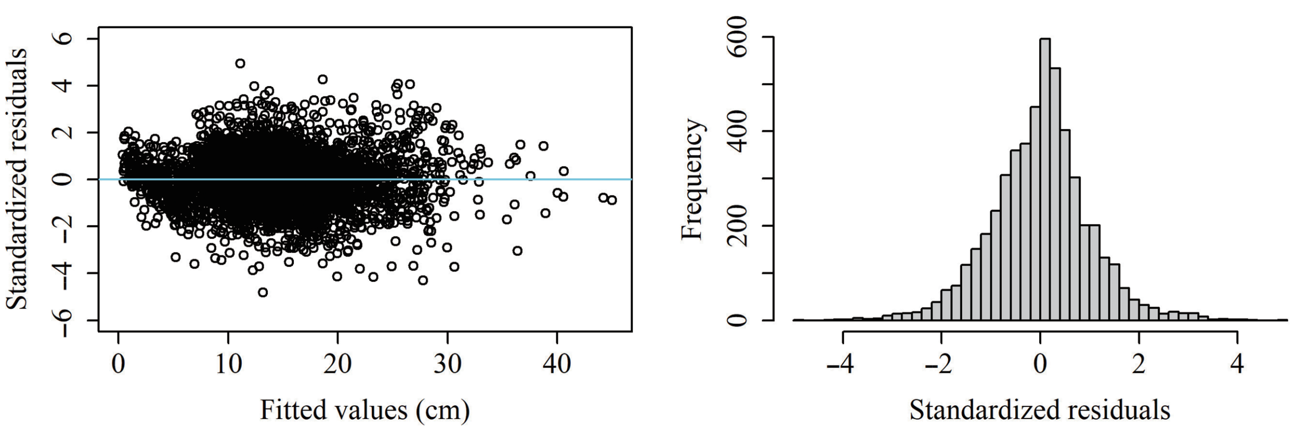

Fig. 2. Scatter plot of standardized residuals vs. fitted diameter overbark values and histogram of standardized residuals on Model K8

Table 2. Summary of goodness-of-fit statistics for taper models

Model	Coefs	$R^{2'}$	RSE (cm)	ΔBIC	Base model
G10	*10*	*0.97913*	*0.7909*	*68*	Eq. 1
G9	*9*	*0.97912*	*0.7912*	*65*	
G8	*8*	*0.97909*	*0.7918*	*65*	
G7	*7*	*0.97880*	*0.7971*	*127*	
G6	6	0.97846	0.8037	204	
G5	5	0.97841	0.8045	207	
G5x	5	0.97826	0.8074	244	
G4	4	0.97748	0.8216	418	
G3	3	0.97665	0.8367	599	
K9	*9*	***0.97936***	***0.7866***	*4*	Eq. 2
K8	*8*	*0.97935*	*0.7869*	***0***	
K7	*7*	*0.97930*	*0.7877*	*3*	
Y9	9	0.96025	0.9325	1,767	Eq. 3
Y8	8	0.95962	0.9388	1,830	
Y7	7	0.95973	0.9395	1,830	
Y6	6	0.95986	0.9406	1,834	
Y5	5	0.95991	0.9467	1,894	
Y3	3	0.95895	0.9597	2,020	

Bold numbers indicate the best model for each statistic.
Italic rows indicate the models selected for cross-validation.
Coefs: number of coefficients in the model
$R^{2'}$: adjusted coefficient of determination
RSE: residual standard error
ΔBIC: Bayesian information criterion (BIC) difference from the lowest BIC = 0 (actual value = 12,293)

hh showed that the mean error by the DBH class was <2.6% of the actual value with sufficient accuracy (PE% within ±1.2%) and good precision. Over 75% of estimations were predicted within 10% error of the actual values, except the smallest DBH class1. For predicted h given d_{ob}, the mean error by RH class was within ±1% of the actual value with sufficient accuracy (PE% within ±1%) and excellent precision in RH classes over 50%. Precision was fair or poor in the lower bole part where the small taper enhanced the ratio of estimated to actual values. The same feature was observed for all tested models.

Table 3. Index values for three aspects and for overall statistics from leave-one-out cross-validation of the seven taper models

Model	d_{ob} given h	h given d_{ob}	v given hh	Overall
G10	1.028	1.050	0.983	1.020
G9	1.028	1.064	0.932	1.008
G8	0.996	1.091	1.068	1.052
G7	1.045	1.181	1.287	1.171
K9	0.967	0.876	0.912	0.918
K8	**0.964**	**0.864**	**0.898**	**0.909**
K7	0.972	0.874	0.92	0.922

Bold numbers indicate the best model for each statistic.
d_{ob}: diameter overbark; h: height; v given hh: log volume overbark given two end heights

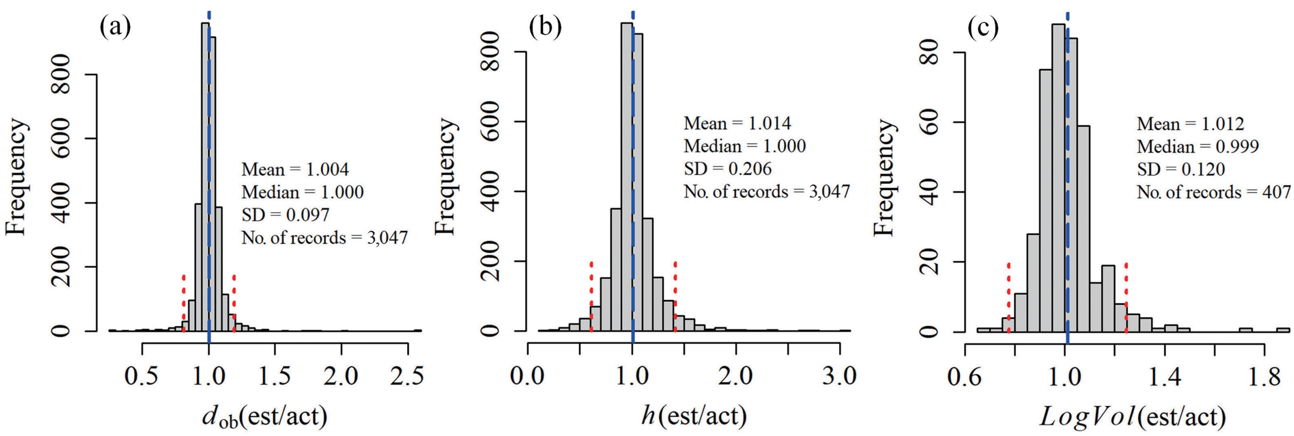

Fig. 3. Leave-one-out cross-validation histograms of Model K8: (a) diameter overbark given height, (b) height given diameter overbark, and (c) log volume overbark given two end heights
The X-axis shows a ratio of the estimated and actual values.
Long dashed lines and dotted lines show mean and mean ± 1.96 SD, respectively.
d_{ob}: diameter overbark; h: height above ground; $LogVol$: log volume overbark; est/act: ratio of estimated to actual values

Table 4. Leave-one-out cross-validation summary for Model K8

Prediction aspect	Class	PE%	RRMSE%	e_{10} (%)	est/act (%)	n
d_{ob} given h	≤10% RH	0.13	4.48	96	99.9±4.5	407
	10% to ≤20% RH	–1.00	4.59	96	101.2±4.4	367
	20% to ≤30% RH	0.20	5.34	94	100.1±5.3	389
	30% to ≤40% RH	0.39	5.41	94	99.9±5.3	390
	40% to ≤50% RH	–0.15	5.85	93	100.5±5.9	391
	50% to ≤60% RH	–0.32	6.66	90	100.9±7.2	385
	60% to ≤70% RH	0.28	8.13	81	100.5±8.3	358
	70% to ≤80% RH	1.47	11.03	58	100.2±12.0	190
	80% to ≤90% RH	–5.15	18.76	40	106.8±19.1	87
	>90% RH	8.55	30.72	39	95.6±39.0	83
h given d_{ob}	≤10% RH	–3.39	37.29	22	104.5±39.8	407
	10% to ≤20% RH	–7.46	25.19	33	107.5±24.1	367
	20% to ≤30% RH	–0.69	22.12	38	100.5±21.6	389
	30% to ≤40% RH	0.72	16.52	50	99.2±16.4	390
	40% to ≤50% RH	–0.03	12.54	62	100.1±12.6	391
	50% to ≤60% RH	0.02	9.32	73	100.1±9.8	385
	60% to ≤70% RH	0.61	7.17	85	99.5±7.5	358
	70% to ≤80% RH	1.12	5.68	94	99.1±5.6	190
	80% to ≤90% RH	–0.79	3.82	99	100.7±3.7	87
	>90% RH	0.69	2.40	100	99.2±2.3	83
v given hh	DBH class1	–0.07	11.03	63	101.7±14.6	81
	DBH class2	–0.06	9.35	81	101.7±10.1	81
	DBH class3	0.66	10.18	75	98.8±9.1	81
	DBH class4	–0.81	8.96	79	102.6±11.9	81
	DBH class5	1.21	10.14	77	101.2±13.4	83

RH: relative height above ground; est/act: ratio of estimated to actual values; e_{10}: number rate of predictions within 10% error of actual value; mean ± SD

2. Bark thickness equation

The $BT2$ samples from the 103 felled trees were used for modeling after removing outliers according to the threshold of three times the median absolute deviation (Leys et al. 2013). The power model equation was obtained from fitting Eq. 5 with the 1,015 $BT2$ samples in this study, as shown in Eq. 8 ($R^{2\prime} = 0.778$).

$$BT2 = 0.322671290389(d_{\mathrm{ob}})^{0.692876496137} \tag{8}$$

where $BT2$ and d_{ob} were measured in centimeters. The coefficients were highly significant ($P < 0.001$). The standard residuals on $BT2$ prediction were evenly distributed and were not heteroscedastic (Fig. 4).

Figure 5 shows a scatter plot of $BT2$ for d_{ob} with the $BT2$ curves predicted by different bark thickness equations. The predicted $BT2$ using our bark thickness equation (Eq. 8) was medium for d_{ob} <~10 cm but thick for d_{ob} >~10 cm relative to that obtained using two other equations (Fig. 5). The FIO-teak1 bark thickness equation predicted thinner $BT2$ than the TPP2 equation.

3. Comparison to the FIO-teak1 equation

Performances of the taper equations with the obtained data (n = 5,183 diameters of 407 trees) indicated that by RH class, the FIO-teak1 equation overestimated stem diameters distinctly with lower precision around the bole part of 10%-60% RH. Similarly, the G5x equation slightly overestimated stem diameters in the 20%-60% RH range (Fig. 6). However, the K8 equation displayed stable and good performance across RH classes.

For response investigation of the taper equations on tree slenderness, the sample tree measurements were divided into four equally populated classes sorted by SC for the taper equations' test on d_{ob} prediction. The SC thresholds were found to be 84.1, 93.0, and 100.7. Compared to SC class1, SC class4 consisted of trees characterized by a high SC value, high stand density, tall

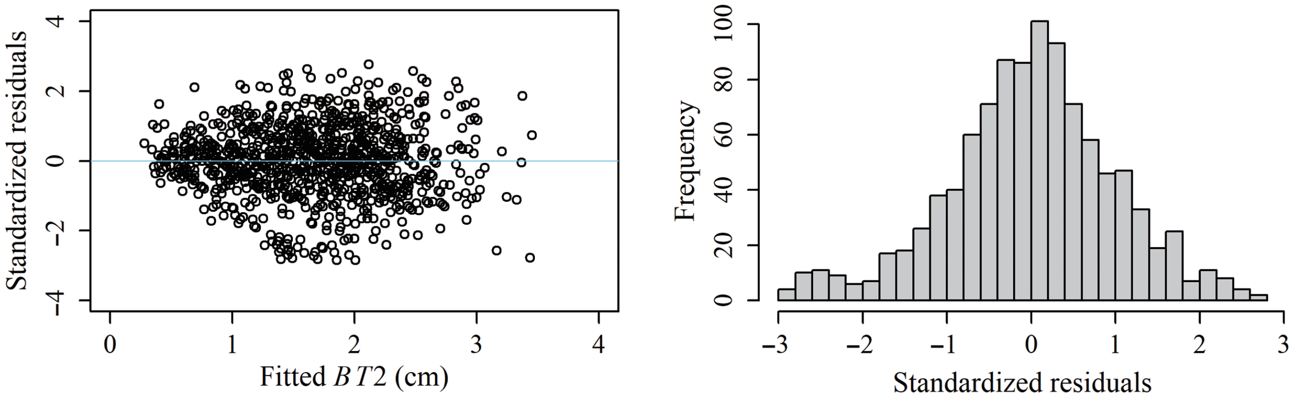

Fig. 4. Scatter plot of standardized residuals vs. fitted double bark thickness (*BT*2) and histogram of standardized residuals on our bark thickness equation

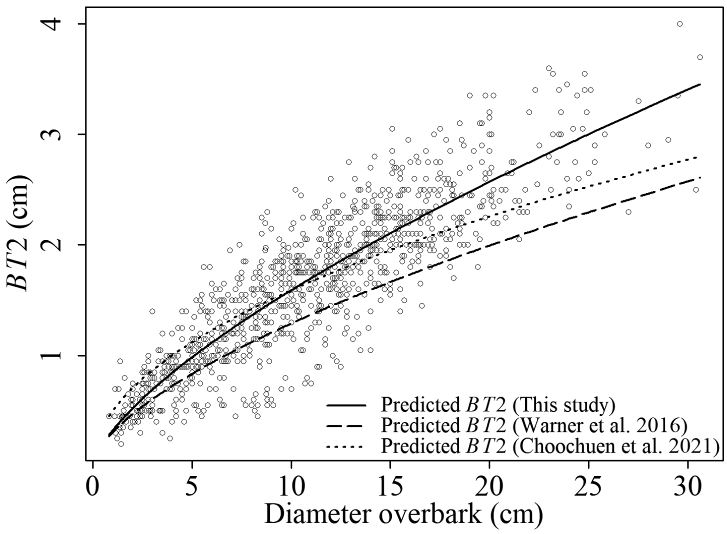

Fig. 5. Scatter plot of double bark thickness (*BT*2) vs. diameter overbark overlaying the *BT*2 curves predicted by the bark thickness equations

tree height, and small DBH but not different age at a 5% significance level (Fig. 7). Figure 8 shows that the d_{ob} overestimation of FIO-teak1 increased for SC class4 with an RH range of 10%-60%. Similarly, the G5x equation overestimated diameters for SC class4 with 30%-60% more RH than the predicted diameters for SC class1. Meanwhile, the K8 equation had the most stable and best performance among the three equations at each RH class for SC class1 and SC class4.

Discussion

Cubic polynomial taper models have been used for stem volume estimation in *Cryptomeria japonica* (Kajihara 1987, Osumi 1959), *Chamaecyparis obtusa* (Kajihara 1987, Maeda 2013), *Larix kaempferi* (Kajihara 1987, Yamada & Ohno 2016), and broad-leaved trees (Hiroshima et al. 2006, Tomita et al. 1991) in Japan. However, in this study, *H* and *D* were significant terms in the second-stage models in Eq. 3, but the Yamada models did not work well for teak plantation trees because, as McTague & Weiskittel (2021) noted about polynomials, the model generated such an oscillating curve as not to estimate the stem diameter suitably (Table 5). This indicates that the stem form of our data might be different from those tree species although comparative studies would determine it.

A plot of height above ground versus predicted

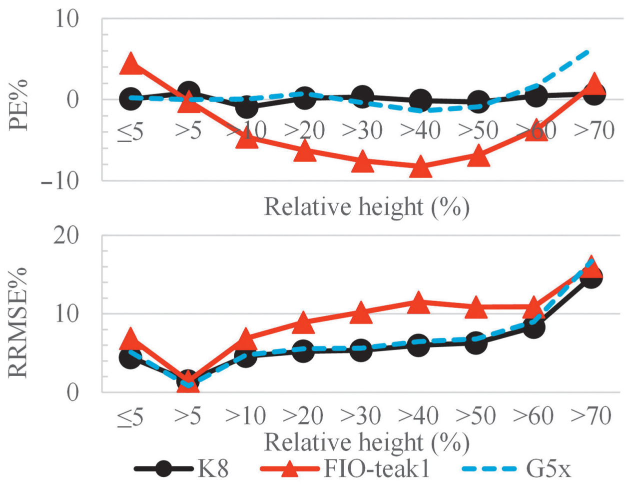

Fig. 6. Evaluation indices by relative height class in stem diameter prediction by the taper equations applied to the dataset (n = 407 trees)

The sample number of each relative height class ranged from 358 to 407.

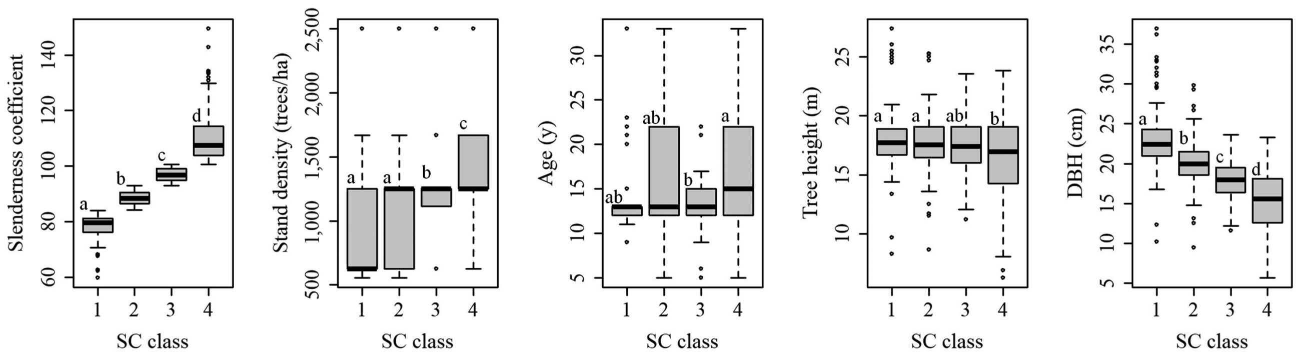

Fig. 7. Characteristics of factors by SC class subset derived from the dataset (n = 407 trees)

n = 102 (SC class1), 102 (SC class2), 100 (SC class3), and 103 (SC class4).
The same alphabet means no significant difference ($P < 0.05$, Scheffe test).
SC: slenderness coefficient of tree; DBH: diameter at breast height overbark

diameter can assist with the comparison between different taper equations based on the stem form of a tree, as shown by Bi & Long (2001) and Choochuen et al. (2021). Three stem profiles were drawn using a taper equation for small, medium, and large trees in our data to display the differences in the predicted stem form associated with changes in tree size (Fig. 9(a), (b), (c)). Differences in stem profile of the K8 versus FIO-teak1 equation (Fig. 9(a)) reflect the combined effect of model features and tree data shown in Figure 9(b) and Figure 9(c), respectively. The tree samples in this study are less paraboloidal at the upper stem and more neiloidal at the lower stem, and these characteristics with become clearer with larger trees (Fig. 9(c)). This was in contrast to FIO plantation trees, as indicated by the differences in the extent of overestimation due to different coefficients between the equations of FIO-teak1 and G5x (Fig. 6). Next, different performances of K8 and G5x based on model features indicated that the model formation of FIO-teak1 could not be applied to our data (Fig. 6). This is graphically denoted by the comparison of stem profiles that the model formation of FIO-teak1 generated a less paraboloidal curve at the upper stem and a slightly inflated curve at the middle stem than Model K8 (Fig. 9(b)). Furthermore, the model formation of FIO-teak1 was limited to slender trees (SC >~100), as indicated by the different performances of the taper equations between SC class1 and SC class4 due to tree

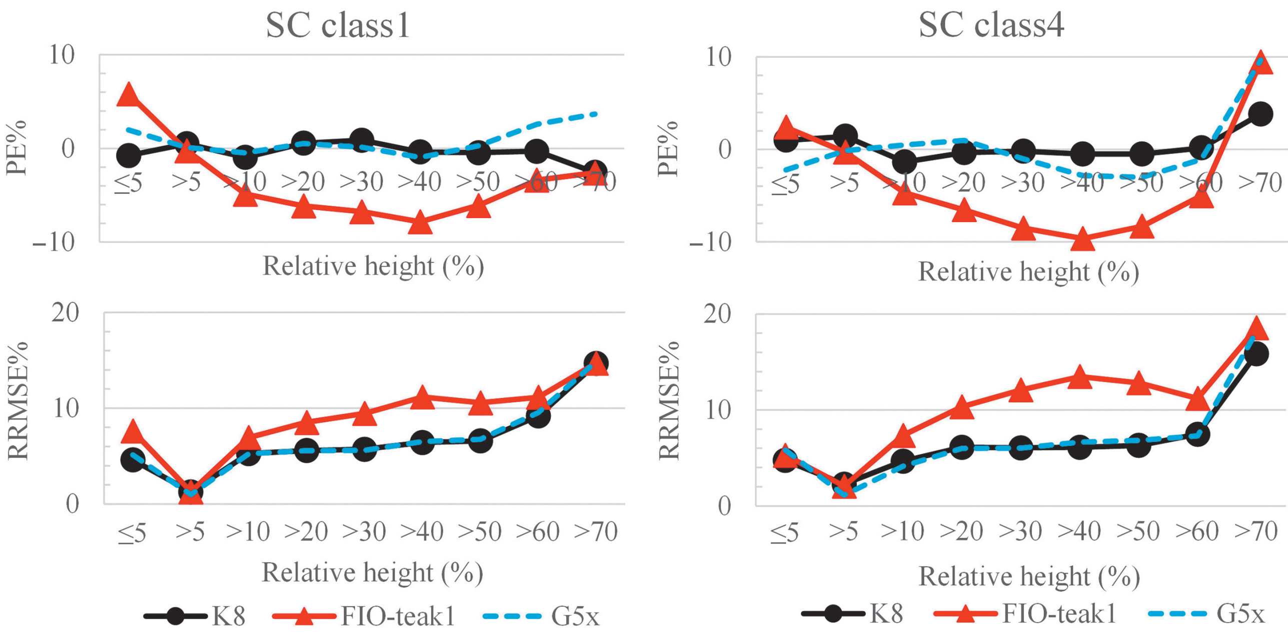

Fig. 8. Differences in stem diameter prediction performances of the taper equations between the subsets of SC class1 and SC class4

The sample number in each relative height class ranged from 103 to 182 in the SC class1 subset and from 90 to 157 in the SC class4 subset.

SC: slenderness coefficient of tree

Table 5. Percentage error from leave-one-out cross-validation appraisal on a prediction of diameter overbark given height for Yamada models

Class	Models						n
	Y9	Y8	Y7	Y6	Y5	Y3	
≤10% RH	7.78	7.87	7.79	7.82	7.84	7.70	407
10% to ≤20% RH	–0.91	–1.02	–1.02	–1.02	–1.04	–0.95	367
20% to ≤30% RH	3.77	3.61	3.64	3.63	3.60	3.85	389
30% to ≤40% RH	6.10	5.96	5.99	5.97	5.95	6.34	390
40% to ≤50% RH	5.67	5.64	5.66	5.65	5.65	6.17	391
50% to ≤60% RH	3.79	3.90	3.88	3.90	3.92	4.53	385
60% to ≤70% RH	1.43	1.60	1.52	1.49	1.58	2.19	358
70% to ≤80% RH	–0.51	–0.64	–0.91	–1.18	–0.99	–0.34	190
80% to ≤90% RH	–11.41	–10.64	–11.67	–11.99	–11.68	–8.91	87
>90% RH	9.04	9.98	9.05	8.88	9.14	11.33	83

RH: relative height

slenderness (Fig. 8). Our sample teak trees were more slender (93.7 ± 13.2 in SC) than the FIO plantation teak trees, which had SC values of 78 (Warner et al. 2016) and 76 (Choochuen et al. 2021). Additionally, our data consisted of various planting distances, and the SC ranged from 80 to 106 and was affected by the planting distance ($P < 0.05$, Kruskal–Wallis test). Thus, we confirmed that the private plantation teak trees were generally slender because of the narrow planting distance and insufficient thinning. Moreover, they show a wide range of slenderness coefficients, probably due to the use of various planting distances.

Model K8, with the same term formation as "Kozak021" of Warner et al. (2016), was the best among all models and tests in our study, contradicting the results reported by Warner et al. (2016), where it almost ranked last. The reason that Kozak Model K8 was the best in our study could be that the term D/H (the ratio of

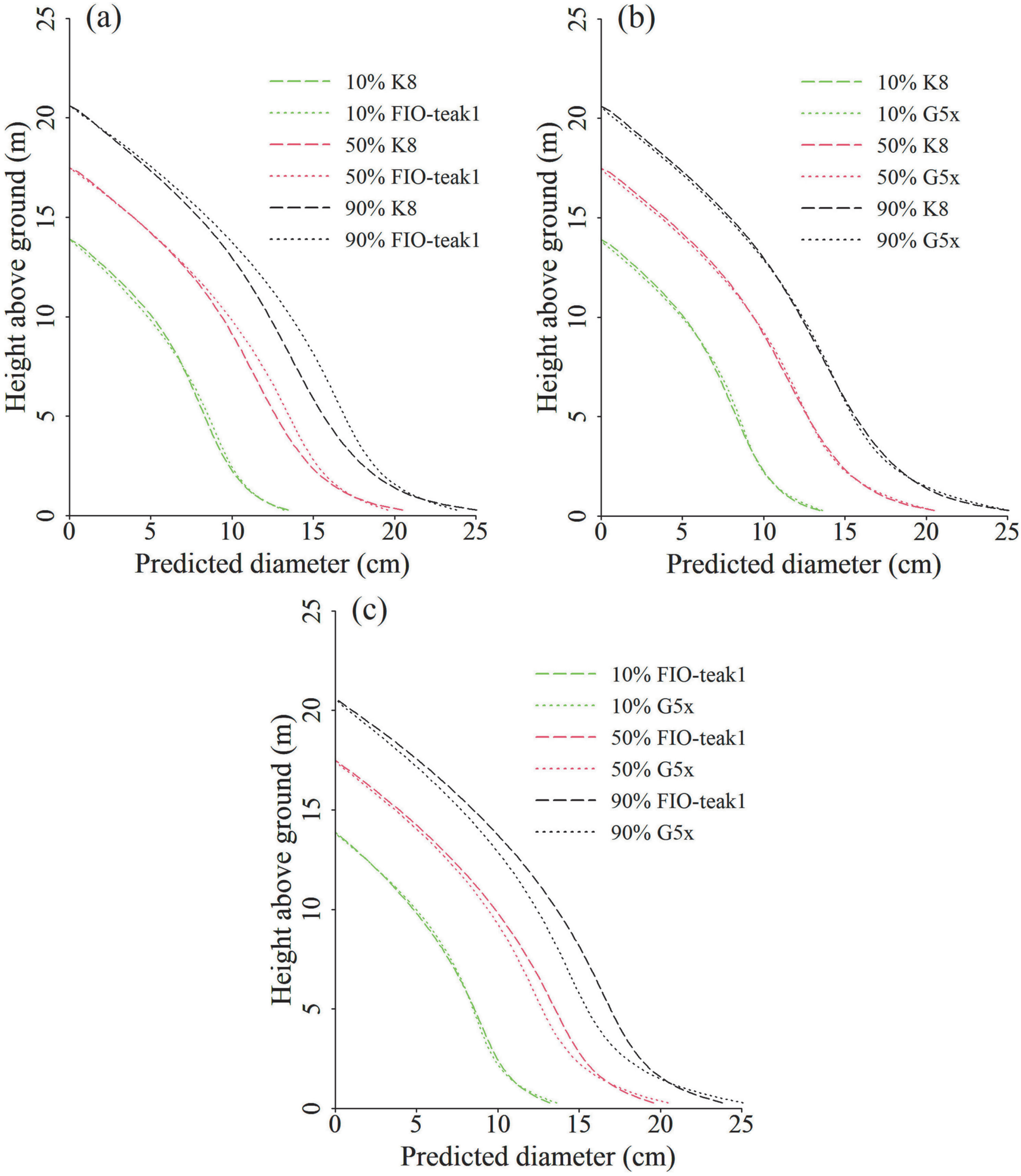

Fig. 9. Stem profiles derived from the taper equations for small (10%), medium (50%), and large (90%) trees: (a) K8 vs. FIO-teak1 equation, (b) K8 vs. G5x equation, and (c) FIO-teak1 vs. G5x equation
%: percentile of DBH

D to *H*) in Model K8 enables the model to adjust to varied tree slenderness because the *D*/*H* ratio represents the live crown ratio affecting stem form (Kozak 1988, Newnham 1988).

Two inferences of the results of applying the taper equations to our dataset were as follows: 1) a tree of our data was different in stem form and slender compared to that of the FIO teak plantation, and 2) the model of FIO-teak1 is unsuitable to approximate stem tree as slender as having SC >~100. Therefore, for private teak plantations in this study area, Model K8 would be better suited than FIO-teak1, and the K8 taper equation with nonzero coefficient values ($P < 0.001$) of a_0 = 1.0974276E+00, a_1 = 1.0369592E+00, a_2 = −7.3704856E−02, b_1 = 4.2266296E−01, b_2 = −1.3398704E−01, b_3 = 4.8976575E−01, b_4 = −1.3386996E+00, and b_5 = 5.0219497E−02 in Eq. 2 is the most suitable.

Choochuen et al. (2021) noted that teak trees in western regions of Thailand tend to have thicker barks than those in northern regions of Thailand. According to Rosell (2016), teak trees can develop thicker barks in the moist environments of the western regions due to processes like transpiration and photosynthesis. However, in the present study, the teak tree barks were thicker than those reported by Warner et al. (2016) and Choochuen et al. (2021) although the sampled teak trees grow under a climate similar to that of the region studied by Warner et al. (2016). Bark thickness variation is associated with the functions of the inner and outer bark, and outer bark thickness is associated with fire protection (Graves et al. 2014, Rosell 2016). Additionally, according to Rosell (2016), bark thickness may reflect an evolutionary response to pressures far beyond fire. Further studies on the different drivers of bark thickness variation are needed to understand bark ecological strategies. Our bark thickness data comprised over 1,000 samples. However, a more extensive dataset could facilitate the estimation of teak bark thickness using multifactorial approaches.

For teak growth, site index indicating forest productivity must be affected by site and management conditions such as soil properties, slope topography, and land use history and fertilization (Noda et al. 2021). In this study, site and management conditions were not considered, but some sites may not be consistent in site index. Subsets by site index in parameter estimation may improve the performance of taper equations because site index can affect stem form (Newberry & Burkhart 1986). Further collection of upper stem diameter and bark thickness data is encouraged for facilitating application of the findings in private teak plantations under different site and management conditions in the country.

Conclusions

GOF analysis narrowed down 18 taper models that included variants of three taper models. Finally, seven selected variants were evaluated using LOO cross-validation. The best-performing model for our dataset was a variant (Model K8) of the Kozak variable-exponent model. Model K8 had sufficiently small errors with mostly excellent precision and provided stable and good predictions across RH classes. Model G5x, having the same formation as “FIO-teak1” and “TPP2,” was less fitting during GOF analysis than the seven selected variants. The private plantation teak trees in the present study were found to be different in stem form and slender, unlike FIO plantation teak trees. The model formation of FIO-teak1 could not fit the slender trees as well as Model K8 could. Therefore, for the private teak plantations in this study area, the FIO-teak1 equation overestimates stem diameters because of unsuitability in model features and differences in stem form, and the K8 taper equation is the most suitable. A comparison of bark thickness measurements suggested that private plantation teak trees generally have thicker barks than FIO plantation teak trees. Our taper and bark thickness model equations are the first to be published for private plantation teak trees in Thailand. They will contribute to the evaluation of the merchantable value of private teak plantations.

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Appendix Table. Nonlinear regression coefficients for the models appraised for goodness-of-fit

a) Goodwin models

Coefficient	G10	G9	G8	G7	G6	G5	G5x	G4	G3
a_0	NA	NA	NA	NA	NA	NA	NA	NA	NA
a_1	2.2994860E+00	2.1521543E+00	2.0601168E+00	2.1469028E+00	2.0720840E+00	2.0797406E+00	1.1051585E+00	1.0256381E+00	1.1617720E+00
a_2	−7.6046475E–02	−6.8401610E–02	−6.3653218E–02	−7.0433154E–02	−5.0392744E–02	−5.0466671E–02	NA	NA	NA
a_3	1.2791635E+00	1.2485250E+00	1.1995433E+00	1.2171568E+00	NA	NA	NA	NA	NA
b_0	−5.1576943E–01	NA	NA	NA	NA	NA	4.5896070E–01	NA	NA
b_1	3.5579868E–02	1.8136756E–02	1.6973606E–02	2.5733657E–02	2.5063179E–02	2.4845886E–02	NA	2.8644051E–02	2.2326944E–02
b_2	5.2468325E+00	1.7735683E+00	2.2905219E+00	NA	NA	NA	NA	NA	NA
c_0	5.3280875E–02	4.1333258E–02	NA	NA	NA	NA	NA	NA	NA
c_1	−5.3235050E–03	−4.7810789E–03	−3.5435530E–03	−3.8173399E–03	−2.0399565E–03	−1.7436381E–03	NA	−6.4352222E–04	NA
c_2	1.7042446E+00	1.7392101E+00	2.0972169E+00	2.1429639E+00	2.0940966E+00	2.0875129E+00	1.6652293E+00	1.5825164E+00	1.4944959E+00
c_3	1.7801631E–02	1.7505375E–02	1.6504638E–02	1.6642698E–02	2.3276636E–03	NA	−9.2417797E–03	NA	NA
c_4	NA	NA	NA	NA	NA	NA	1.7527397E–03	NA	NA

b) Kozak models

Coefficient	K9	K8	K7
a_0	1.0854821E+00	1.0974276E+00	1.0752663E+00
a_1	1.0365442E+00	1.0369592E+00	1.0243044E+00
a_2	−6.9053025E–02	−7.3704856E–02	−5.3526278E–02
b_1	4.4831942E–01	4.2266296E–01	4.1890034E–01
b_2	−9.9320167E–02	−1.3398704E–01	NA
b_3	4.8387422E–01	4.8976575E–01	4.6147900E–01
b_4	−1.5790274E+00	−1.3386996E+00	−1.7149072E+00
b_5	4.1921954E–02	5.0219497E–02	5.0705936E–02
b_6	5.7518611E–02	NA	NA

c) Yamada models

Coefficient	Y9	Y8	Y7	Y6	Y5	Y3
a_1	1.7698774E+00	2.1075355E+00	2.6486449E+00	2.6413423E+00	2.6325752E+00	2.6316325E+00
a_2	1.4610733E–01	2.9237619E–02	NA	NA	NA	NA
a_3	−8.5261170E–02	NA	NA	NA	NA	NA
b_1	−2.9904929E+00	−3.4686503E+00	−4.2295599E+00	−4.3215023E+00	−4.3103344E+00	−4.0013873E+00
b_2	−2.1829298E–01	−5.3496384E–02	−1.2209262E–02	NA	NA	NA
b_3	1.4016498E–01	2.0162017E–02	2.0041062E–02	1.4196606E–02	1.4271391E–02	NA
c_1	2.3585759E+00	2.5157900E+00	2.7630030E+00	2.8387673E+00	2.8778633E+00	2.6325779E+00
c_2	8.1770658E–02	2.7934693E–02	1.4429697E–02	4.9922201E–03	NA	NA
c_3	−5.7384603E–02	−1.8295437E–02	−1.8194203E–02	−1.3682616E–02	−1.1351228E–02	NA

Shaded coefficients are highly significant ($P < 0.001$) based on the nonlinear regression. NA: not applicable