REVIEW
Positive Mathematical Programming for Farm Planning: Review

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Abstract

In the field of farm management and in related multidisciplinary fields such as bio-economic farm modeling and hydro-economic regional modeling, much attention has recently been paid to positive mathematical programming (PMP), primarily because of its ability to exactly reproduce an observed set of endogenous input variables in the model (e.g., an observed land-use pattern for crop productions) as the result of optimization. In mathematical terms, PMP is an inverse problem of quadratic programming (QP), where the objective function is calibrated on the basis of the Kuhn-Tucker conditions for the optimization of the QP model and of a linear programming model that is prepared for the calibration of parameters in the QP model. The two types of optimum conditions derived from the models are combined to obtain linear equations for the calibration of the QP model. However, as is often the case with inverse problems, the equations for the calibration are indefinite because the number of parameters to be calibrated surpasses the number of equations. As a result, various methods have been proposed to solve this so-called "ill-posed" problem. The main objectives of the present paper are to examine how the calibration methods developed in previous PMP models are related to one another and to propose practical procedures for determining which calibration method is the most appropriate from the viewpoint of sensitivity analyses. A simple conceptual framework is proposed to relate the previously developed calibration methods, and it is then applied to exemplify criteria for selecting a calibration method from the viewpoint of simulation results. A new direction in PMP-based farm modeling in which more feasible simulation results can be derived is also discussed.

Discipline: Agricultural economics
Additional key words: inverse problem, ill-posed problem, calibration selection, feasible simulation result, bio-economic farm model

Introduction

Because of growing concern about environmental pollution caused by economic activities including the exploitation of fossil fuels and intensive agricultural land use, a number of multidisciplinary research projects have been launched worldwide to quantify the trade-offs between the economic and environmental impacts of agricultural/industrial production. The goal of these projects is to help policymakers design policy measures that will lead to the creation of better production systems that balance economic prosperity with environmental conservation. In such attempts, the choice of an economic model of production often determines the direction and the limi-

tation of projects, and that choice is therefore a matter of significant concern to modelers. Activity-based mathematical programming (MP) is a popular way of modeling the economics of production, as widely observed in the literature of farm management[10] and in related fields such as bio-economic farm modeling[20] and hydro-economic regional modeling[39]. The activity analysis is popular for two primary reasons. One is its ability to disaggregate an overall system of crop productions into individual crop production systems. The disaggregation function of production systems allows the linkage of a variety of environmental assessment tools (e.g., a life-cycle inventory analysis) with individual crop production[29]. The other reason is the ability to optimize the disaggregated crop production systems at a particular analytical scale (e.g., at

a farm scale) while taking producers' behavioral hypotheses and economic, technical, and institutional constraints as a whole into consideration. Optimization under the initial condition (i.e., static analysis) shows not only the best possible combination of endogenous input variables such as the land areas allocated among crop productions, but also the economic profit and environmental load resulting from the production activities. Subsequent optimizations under different conditions (i.e., comparative statics analysis) indicate how the initially optimized area allocation and the resultant economic gain and environmental load change in response to changes in exogenous variables, for example, changes in crop prices, crop varieties, and government subsidies. This type of analysis provides a theoretical framework for policymakers and academics to examine the interactions among economic gains, environmental loads, and public expenses involved with a farming system as well as to design policy measures that could improve the farming system from the viewpoint of social welfare[29].

Despite these theoretical strengths, however, traditional MP has a weakness. As has been frequently discussed in the context of linear programming (LP)[16, 17], traditional MP does not generally reproduce the observed set of endogenous input variables (e.g., the observed land-use pattern for crop productions) in the static analysis. The economic profit and environmental load evaluated at the theoretical optimum point therefore deviate from their observed values, meaning that the traditional approach does not adequately reproduce the very starting point in the simulation of trade-offs between the economic and environmental impacts of production. In the traditional farm modeling, the gap between the theoretically optimized economic gain and the observed value has been interpreted as an index of economic inefficiency in the farming system, and the theoretical result on the area allocation was regarded as the ultimate goal producers should strive to reduce the inefficiency in the system, which is why traditional MP is referred to as a normative MP[4]. Although this interpretation is convenient for discussing the potential of a farming system and a direction for its economic improvement, the reliability of such an analysis heavily depends on whether observed producer behaviors are properly captured in the model. Unfortunately, a normative MP does not always play the role, so a number of trial-and-error methods have been applied to minimize the gap between the theoretically optimized area allocation and the observed allocation. One common approach has been to impose as many economic, technical, and institutional constraints as possible on the MP model. This approach has contributed to excluding some unrealistic solutions from the feasible set of endogenous variables; however, it has not always worked satisfactorily, in part because of a lack of data availability.

To bridge the gap between the theoretically optimized values and the observed values of endogenous input variables in MP model, a great deal of attention has been paid in the last decade to a more systematic approach called positive mathematical programming (PMP). PMP ensures that an MP model exactly reproduces an observed set of endogenous input variables in the model as the result of static analysis. The exact reproduction of the given reference point is normally achieved using quadratic programming (QP), where the objective function is calibrated on the basis of the Kuhn-Tucker conditions for the optimization of the QP model and of an LP model that is prepared for the calibration of parameters in the QP model. The two types of optimum conditions derived from the models are combined to derive linear equations for the calibration; however, the derived equations are indefinite because the number of parameters to be calibrated surpasses the number of equations. As a result, various methods have been proposed to solve this so-called ill-posed problem[32]. The methods of Howitt and Mean[18], Paris[31], Howitt[16, 17], Paris and Howitt[32], Helming, Peeters and Veendendaal[15], Preckel, Harrington and Dubman[34], Röhm and Dabbert[35], and Heckelei and Wolff[13] are all prominent examples.

The objective of the present paper is to briefly survey the PMP studies that have been published. Although several survey papers have been published on this subject (e.g., de Frahan et al.[6]), their main focus was on individual calibration methods; that is, the authors studied how the ill-posed problem has been solved. The focus of the present study is on two methodological issues: how the calibration methods developed in previous PMP studies are related to one another and which method is the most appropriate from the viewpoint of comparative statics. As mentioned above, the ill-posed problem has yielded a number of calibration methods that reproduce a given reference point as the result of static analysis; however, these methods could yield different simulation results in terms of both economic and environmental aspects[29]; that is, previous PMP models do not always ensure a "positive" result when it comes to comparative statics. To ensure the derivation of feasible simulation results in addition to the reproduction of a given reference point, it is necessary to establish criteria for selecting a calibration method. In the present paper, after a brief introduction to PMP, a simple framework is proposed to relate the previously developed calibration methods, especially those of Howitt and Mean[18], Paris[31], and Howitt[16, 17]. The framework is then applied to exemplify practical criteria for selecting a calibration method from the viewpoint of comparative

statics. A new direction for PMP-based farm modeling in which more feasible comparative statics can be derived is also discussed.

Background

In mathematical terms, PMP is an inverse problem of QP model[23], where the objective function is calibrated on the basis of the Kuhn-Tucker conditions for the optimization of the QP model with an observed set of endogenous input variables such as the land areas allocated among crops. In this paper, the quadratic terms of the objective function are attributed to the nonlinearities in the accounting cost for agricultural materials such as fertilizers and biocides that can be purchased in markets, and the producer is assumed to be a price-taker who pursues net profit maximization as described below (see Table 1 for the definition of variables and parameters):

$$\max_{\mathbf{x}} \pi(\mathbf{x}) : \pi(\mathbf{x}) \equiv \mathbf{p}'\mathbf{x} - \left\{ \mathbf{d}'\mathbf{x} + 0.5\mathbf{x}'\mathbf{Q}\mathbf{x} \right\} \quad (1)$$

$$s.t. \ \mathbf{Ax} \leq \mathbf{b}, \quad (2)$$

$$\mathbf{x} \geq \mathbf{0}. \quad (3)$$

The function $C(\mathbf{x}) \equiv \mathbf{d}'\mathbf{x} + 0.5\mathbf{x}'\mathbf{Q}\mathbf{x}$ denotes the accounting cost (hereafter referred to as the "cost function") and $\mathbf{Q}$ is symmetric because of Young's theorem and is assumed to be positive semi-definite (in that $\mathbf{x}'\mathbf{Q}\mathbf{x} \geq 0$ for all $\mathbf{x}$). Parameters $\mathbf{p}$, $\mathbf{A}$, and $\mathbf{b}$ are specified on the basis of farm

management data (see Nakashima[29] for the specification procedure frequently applied in farm management); once specified, they are denoted as $\hat{\mathbf{p}}$, $\hat{\mathbf{A}}$, and $\hat{\mathbf{b}}$, respectively. $\mathbf{d}$ and $\mathbf{Q}$ are the parameters to be calibrated. For their calibration, standard PMP employs the following LP model (see Table 1 for the definition of variables and parameters):

$$\max_{\mathbf{x}} \pi(\mathbf{x}) : \pi(\mathbf{x}) \equiv \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} \quad (4)$$

$$s.t. \ \mathbf{Ax} \leq \mathbf{b}, \quad (5)$$

$$\mathbf{x} \leq \mathbf{x}^{\mathbf{o}} + \boldsymbol{\varepsilon}, \quad (6)$$

$$\mathbf{x} \geq \mathbf{0}, \quad (7)$$

where equation (6) is a calibration constraint. Parameters $\mathbf{p}$, $\mathbf{A}$, and $\mathbf{b}$ in the LP model (4)–(7) are specified in the same way as they are in the QP model (1)–(3). Parameter $\mathbf{c}$ is specified on the basis of farm management data (see Nakashima[29] for the procedure), and once specified, it is denoted as $\hat{\mathbf{c}}$. Given that the optimum dual variables associated with resource constraints coincide both in the QP model (1)–(3) and the LP model (4)–(7) (i.e., $\boldsymbol{\theta}^* = \boldsymbol{\lambda}^*$), then the sufficient and necessary conditions for the optimization of the models derive linear equations for the calibration of $\mathbf{d}$ and $\mathbf{Q}$, such that

$$\hat{\mathbf{c}} + \boldsymbol{\rho}^* = \mathbf{d} + \mathbf{Q}\mathbf{x}^{\mathbf{o}}. \quad (8)$$

Table 1. Definition of variables and parameters

Variables	
$\mathbf{x}$	an ($n \times 1$) vector of primal variables that are defined as land area allocated to each crop production [LP, QP]
$\boldsymbol{\lambda}$	an ($m \times 1$) vector of dual variables associated with fixed but allocatable resource constraints [LP] where the number of resources needs to be fewer than that of primal variables, i.e., $n > m$.
$\boldsymbol{\theta}$	an ($m \times 1$) vector of dual variables associated with resource constraints [QP]
$\boldsymbol{\rho}$	an ($n \times 1$) vector of dual variables associated with calibration constraints [LP]

Parameters	
$\mathbf{p}$	an ($n \times 1$) vector of revenues per unit area [LP, QP]
$\mathbf{c}$	an ($n \times 1$) vector of accounting costs per unit area [LP]
$\mathbf{A}$	an ($m \times n$) matrix of input/output coefficients [LP, QP]
$\mathbf{b}$	an ($m \times 1$) vector of resource constraints, which is set as $\mathbf{b} = \mathbf{Ax}^{\mathbf{o}}$ [LP, QP]
$\mathbf{d}$	an ($n \times 1$) vector of linear cost coefficients to be calibrated [QP]
$\mathbf{Q}$	an ($n \times n$) symmetric and positive semi-definite matrix of quadratic cost coefficients to be calibrated [QP]

Observations and etc.	
$\mathbf{x}^{\mathbf{o}}$	an ($n \times 1$) vector of observed primal variables (i.e., the reference point) [LP, QP] They are positive by nature.
$\boldsymbol{\varepsilon}$	an ($n \times 1$) vector of small positive numbers [LP]

Equation (8) indicates how parameters **d** and **Q** should be calibrated so that the QP model (1)–(3) reproduces the reference point **x**^{o} as the result of optimization. The problem is that, without additional information, equation (8) does not have a unique solution in **d** and **Q** because there are fewer equations than unknowns to be calibrated (i.e., $n < n + 0.5n (n + 1)$), which is why this type of calibration is referred to as an ill-posed problem[32].

To solve the ill-posed problem, the following assumption has frequently been imposed:

Assumption 1: Matrix **Q** is diagonal; that is, $q_{ij} = 0$ if $i \neq j$, where q_{ij} $(i = 1,2,\cdots, n; j = 1,2,\cdots, n)$ are the elements of **Q**.

Assumption 1 reduces equation (8) to

$$\hat{c}_i + \rho_i^* = d_i + q_{ii}x_i^o \ (i = 1, 2, \cdots, n), \tag{9}$$

where $\hat{c}_i, \rho_i^*, d_i$, and x_i^o $(i = 1,2,\cdots, n)$ denote the elements of $\hat{\mathbf{c}}, \boldsymbol{\rho}^*, \mathbf{d}$, and $\mathbf{x}^o$, respectively. At this stage, the number of equations is still less than the number of parameters to be calibrated (i.e., $n < 2n$), so Howitt and Mean[18] and Paris[31] imposed the following additional assumptions, respectively.

Assumption 2.1 (Howitt and Mean[18]): $d_i = \hat{c}_i$ $(i = 1,2,\cdots, n)$.

Assumption 2.2 (Paris[31]): $d_i = 0$ $(i = 1,2,\cdots, n)$.

Assumptions 2.1 and 2.2 reduce the number of equations to match the number of parameters to be calibrated; therefore, respective unique solutions can be derived from equation (9) as

$$\text{Assumption } 2.1 \Rightarrow q_{ii} = \frac{\rho_i^*}{x_i^o} \ (i = 1, 2, \cdots, n), \tag{10}$$

$$\text{Assumption } 2.2 \Rightarrow q_{ii} = \frac{\hat{c}_i + \rho_i^*}{x_i^o} \ (i = 1, 2, \cdots, n). \tag{11}$$

To solve equation (9) in an alternative way, Howitt[17] took into consideration the relation of the cost function $C(\mathbf{x}) = \sum_{i=1}^{n} (d_i x_i + 0.5 q_{ii} x_i^2)$ to the average cost data for each activity (i.e., $\hat{c}_i$) such that:

$$\begin{aligned} &\hat{c}_i x_i^o = d_i x_i^o + 0.5 q_{ii} \left\{ x_i^o \right\}^2 \\ &\Leftrightarrow \hat{c}_i = d_i + 0.5 q_{ii} x_i^o \ (i = 1, 2, \cdots, n). \end{aligned} \tag{12}$$

Because of equation (12), the number of equations matches that of the parameters to be calibrated (i.e., $2n$). In addition, the coefficient matrix of equations (9) and (12), that is,

$\begin{bmatrix} 1 & x_i^o \\ 1 & 0.5x_i^o \end{bmatrix}$, is nonsingular because of the nature of the reference point (i.e., $x_i^o \neq 0$). Therefore, the parameter set of d_i and q_{ii} is uniquely derived as

$$d_i = \hat{c}_i - \rho_i^* \ (i = 1, 2, \cdots, n), \tag{13}$$

$$q_{ii} = \frac{2\rho_i^*}{x_i^o} \ (i = 1, 2, \cdots, n). \tag{14}$$

Discussion

In the present study, the three calibration methods summarized in the previous section are classified into two generations. The following sections describe the classification with some additional characteristics of the calibration methods, and propose a simple framework for relating the calibration methods to one another. The framework is then applied to discuss a new direction for PMP-based farm modeling studies.

1. The first generation

The first generation of calibration methods is represented by the methods of Howitt and Mean[18] and Paris[31]. Despite their differences—the former explains the average cost data $\hat{c}_i$ by the linear term of the quadratic cost function (i.e., d_i) and the latter explains it by the quadratic term (i.e., q_{ii})—the present study categorized them in the same generation because they use similar procedures to solve equation (9). They both imposed an ex-ante restriction on the cost function a priori (Assumption 2.1 or 2.2) to reduce the number of parameters to be calibrated to match the number of calibration equations. In other words, the space that parameter d_i and q_{ii} could take was narrowed by providing additional assumed information to the extent that a unique pair (d_i, q_{ii}) remained.

Although the approach is probably the easiest way to narrow the parameter space (d_i, q_{ii}), the assumptions imposed by them are just two of many possible options to solve equation (9), which may be illustrated by solving equation (9) as a problem of allocating the left-hand terms (i.e., $\hat{c}_i$ and ρ_i^*) to the right-hand terms (i.e., d_i and $q_{ii} x_i^o$) such that:

$$d_i = s_i \hat{c}_i + t_i \rho_i^* \ (i = 1, 2, \cdots, n), \tag{15}$$

$$\begin{aligned} q_{ii} x_i^o &= (1 - s_i)\hat{c}_i + (1 - t_i)\rho_i^* \\ \Leftrightarrow q_{ii} &= \frac{(1 - s_i)\hat{c}_i + (1 - t_i)\rho_i^*}{x_i^o} \ (i = 1, 2, \cdots, n), \end{aligned} \tag{16}$$

where s_i and t_i denote the respective allocation rate of $\hat{c}_i$ and ρ_i^* to d_i, and where the positive semi-definitiveness

of matrix $\mathbf{Q}$ restricts s_i and t_i to $t_i \leq -\dfrac{\hat{c}_i}{\rho_i^*} s_i + \dfrac{\hat{c}_i}{\rho_i^*} + 1$

when $\rho_i^* > 0$ (Fig. 1) and $s_i \leq 1$ when $\rho_i^* = 0$ (Fig. 2). In the framework, Assumptions 2.1 and 2.2 are located at point A $(s_i, t_i) = (1, 0)$ and point B $(s_i, t_i) = (0, 0)$ $(i = 1,2,\cdots, n)$, respectively (Fig. 1); that is, substituting points A and B for equation (15) derives Assumptions 2.1 and 2.2, respectively. Likewise, the substitution of an arbitrary feasible point on the s_i–t_i axis (except points A and B) for equation (15) derives an alternative to Assumptions 2.1 and 2.2, and the substitution of the alternative assumption for equation (9) derives another pair (d_i, q_{ii}) that solves equation (9).

In this manner, one can obtain numerous parameter sets (d_i, q_{ii}) that solve equation (9), as summarized in equations (15) and (16). The problem with this type of calibration is, however, that one does not always have a sound reason for selecting a particular set (s_i, t_i) to decide (d_i, q_{ii}). It is therefore difficult to judge which calibration method is superior to another, at least at the static analysis stage, because all of the pairs (d_i, q_{ii}) derived through
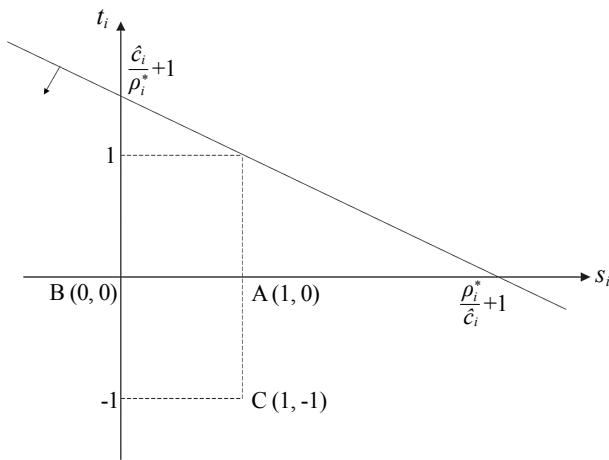

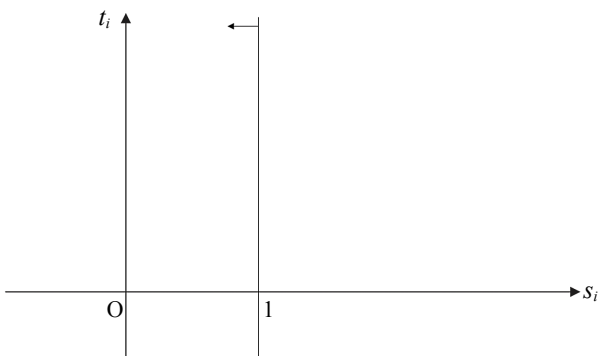**Fig. 1. The domain of s_i and t_i when $\rho_i^* > 0$**

Fig. 2. The domain of s_i and t_i when $\rho_i^* = 0$

the above-mentioned procedure ensure that the QP model (1)–(3) reproduces a given reference point x_i^o as the result of optimization. As a consequence, application of the first generation of PMP to farming systems has been rather limited (see Júdez et al.[22, 23], Umstätter[37], and Wade, Whitehead and O'Shea[38] for the application of Paris's method[31] and Iglesias and Blanco[19] for the application of Howitt and Mean's method[18]).

2. The second generation

It was Howitt's[17] calibration method that drastically increased the application of PMP to farming systems (his seminal paper[16] has been cited no less than 130 times according to the Web of Science®). The calibration method, the result of which is shown in equations (13) and (14), can be interpreted in the same manner as the methods of Howitt and Mean[18] and Paris[31]—the calibration result can be located at point C $(s_i, t_i) = (1,-1)$ $(i = 1,2,\cdots, n)$ in Fig. 1; that is, the substitution of point C for equation (15) derives equation (13) as an alternative to Assumptions 2.1 and 2.2, and the substitution of equation (13) for equation (9) derives equation (14). However, point C was not selected by Howitt's method because equation (13) was assumed a priori, but because the average cost data $\hat{c}_i$ was taken into consideration in the form of equation (12). Whereas the first generation of PMP relied on additional information provided as an ad hoc assumption to select a point in the s_i–t_i axis, Howitt's method exploited the additional observed information, i.e., the empirical relationship formulated by equation (12). The relationship of average cost data to the cost function increased the number of independent equations for calibration to match the number of parameters to be calibrated, thereby narrowing the parameter space (d_i, q_{ii}) to the extent that the unique pair remained, as shown in equations (13) and (14). Because of this change, the present study categorized Howitt's method into the second generation. Subsequent studies such as Helming[14], Helming, Peeters, and Veendendaal[15], Preckel, Harrington, and Dubman[34], and Röhm and Dabbert[35] adopted alternative types of empirical relationships with the QP model to propose variants of Howitt's method. The calibration methods proposed in these studies also represent the second generation of PMP studies.

3. Transition to the second generation

The calibration strategy employed in Howitt's[17] method provided an empirical basis for selecting a particular point on the s_i–t_i axis (Figs. 1 and 2). In addition, the introduction of equation (12) to equation (9) allowed several desirable properties to be derived.

First, the method ensures not only the reproduction

of a given reference point x_i^o but also the reproduction of a given accounting cost $\hat{c}_i x_i^o$ and profit $(\hat{p}_i - \hat{c}_i) x_i^o$ for each production process. These properties are not guaranteed without the use of equation (12). For example, the calibration method of Howitt and Mean[18], whose result is shown in Assumption 2.1 and equation (10), derives the accounting cost as $(\hat{c}_i + 0.5\rho_i^*) x_i^o$, and the method of Paris[31], whose result is shown in Assumption 2.2 and equation (11), derives the accounting cost as $0.5(\hat{c}_i + \rho_i^*) x_i^o$. The former overestimates the observed cost and underestimates the observed profit by $0.5\rho_i^* x_i^o$ except when $\rho_i^* = 0$, whereas the latter overestimates the observed cost and underestimates the observed profit by $0.5|\hat{c}_i - \rho_i^*| x_i^o$ when $\rho_i^* > \hat{c}_i$ (the direction of the deviation is opposite when $\rho_i^* < \hat{c}_i$).

Second, Howitt's[17] method excels in tractability in the sense that the utilization of equation (12) requires no additional data collection. In fact, the dataset required for the execution of Howitt's[17] method is exactly the same as those used in the first-generation methods because, except for a given reference point x_i^o, equation (12) only requires the average cost data ($\hat{c}_i$), which is also essential for the specification of the LP model (4)–(7) commonly used in standard PMP. Other calibration methods in the second generation require some additional data collection or estimation. For example, Helming, Peeters, and Veendendaal[15] and Helming[14] proposed a calibration method with the use of additional information on the supply elasticity; however, it is unlikely that such information is available at no additional cost (see Medellín-Azuara[27] and Medellín-Azuara et al.[28] for applications of this method).

As a consequence of its advantages, Howitt's[17] method and a variant[16] of it accelerated the application of PMP to farming systems (the variant employed an empirical relationship of average yield data with the yield function; see Howitt[16]). Such applications can be widely observed in the field of farm management and in related fields such as bio-economic farm modeling[20] and hydro-economic regional modeling[39]. The scope of analysis ranges from the farm level[1, 2, 8, 29, 30] to the regional and interregional level[5, 7, 9, 21, 24, 25, 26, 33, 36] to the national level[11]. In the majority of applications, the quadratic terms of the objective function were attributed, as in the present paper, to the linearly increasing average cost function, and the rest of the studies often attributed the terms to the linearly decreasing average yield function[2, 16, 30, 40]. Producers have been assumed to maximize net profits in most cases, but it would also be worthwhile to take other aspects of producer behavior into consideration, for example, their attitudes towards price/yield risk and uncertainties that prevail in the agricultural sector, as well as to enlarge the analytical framework by employing multiobjective criteria so that the interactions among individual objectives could be analyzed. It is also important to continue the effort to generalize the representation of production technologies[7, 17, 35].

4. The third generation and a new direction for PMP-based farming system models

As the second-generation variants of Howitt's method[17] developed, ambiguity arose because there were as many solutions to equation (9) as there were ways of relating the QP model (1)–(3) to economic and technical observations[14, 15, 34, 35]. A practical solution to this problem is to select a calibration method that is suitable for available dataset, which is one of the primary reasons why Howitt's[17] calibration method has been so popular. In the period during which Howitt's method flourished in PMP applications, an attempt was made to introduce a statistical criterion called entropy maximization to the calibration[13, 32]. Models employing this entropy-based calibration composed the third generation of PMP, and several were applied to farming systems[3, 12]. This latest generation can be interpreted as providing a statistical solution to the ambiguity involved in choosing a calibration method in the first- and second-generation models. This latest generation also enabled the employment of multiple reference points rather than a single reference point[4], allowing to narrow the gap between the previous PMP generations and econometric approaches that deal with a large number of observations[13].

Although PMP studies have contributed to reproducing a given reference point (or approximating reference points) from the three generations, the contributions have been limited to a static analysis. Little attention has been paid to the fact that the calibrated models can derive different simulation results for both the economic and environmental aspects[29]; in other words, the previous PMP models do not always ensure a "positive" result when it comes to comparative statics. This is a serious problem especially when quantifying the trade-offs between the economic and environmental impacts of agricultural production. To improve the quantification of the trade-offs between the economic and environmental impacts of agricultural production, it is necessary to establish a method of deriving feasible simulation results in addition to the reproduction of a given reference point. One possible approach is to select a calibration method from the candidates that satisfy equation (9), and the framework indicated by equations (15) and (16) might be of some help in this regard. A calibration method could be selected using the following three-step procedure. The first step is to calibrate the QP model (1)–(3) with the method defined by each feasible point (s_i, t_i) in Figs. 1 and 2. Because it is

practically difficult to conduct the calculations for all feasible points (s_i, t_i), the range might need to be restricted, for example, to the neighborhood around point C $(s_i, t_i) = (1, -1)$, which corresponds to Howitt's method[17] that has the several desirable properties as already discussed above. The second step is to perform a comparative statics analysis on the calibrated models to derive simulation results in response to the changes in exogenous variables, for example, the changes in output prices and resource constraints, as experienced by a producer. The final step is to select a calibration method on the basis of the comparative statics results. An unsophisticated but practical way of selecting a calibration method from the candidates is to consult with stakeholders such as farmers, policymakers, and academics and have them select the most feasible simulation results among the candidates. Doing so would enable modelers to obtain a calibration method that ensures the derivation of feasible simulation results as well as the reproduction of a given reference point. It would also be interesting to verify whether the stakeholders support the results derived from Howitt's method[17] from the viewpoint of comparative statics. The final step of calibration selection could be more sophisticated if modelers could access the database of endogenous input variables, such as the land areas allocated among crops, over multiple production periods. Assuming that the function to be calibrated (i.e., the cost function in the present paper) does not shift during the period of analysis, the calibration selection could be translated into minimizing such criteria as the sum of the absolute deviations between the observed and the optimum crop area allocations[1], $\sum_{i=1}^{n} \sum_{k=1}^{l} |x_{ik}^* - x_{ik}^o|$, and the sum of the squares of deviations between the observed and the optimum crop area allocations, $\sum_{i=1}^{n} \sum_{k=1}^{l} (x_{ik}^* - x_{ik}^o)^2$, where k stands for the production year (the base year of the reference point is set as $k = 0$). Selecting a calibration method in the light of the criteria could develop a more robust PMP-based farming system model, but verification with empirical data remains the subject of future research.

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