Application of Risk Programming Method to Forest Planning

By YASUAKI KUROKAWA

Kansai Branch, Forestry and Forest Products Research Institute
(Momoyama, Fushimi, Kyoto, 612 Japan)

Forest production, which requires an extremely long period of time like several decades, is greatly dominated by natural forces, as compared with other production activities. Forest management is always exposed to instability due to difficulties of forecasting natural factors affecting forest production, and of gripping exactly the effects of natural factors and market price of the products over a long period of time in the future. Thus, forest owners are generally characterized by their averted attitude for instability, and, therefore, it is required to establish a forest planning method, in which the instability is taken into account. In this paper, a basic method of such a forest planning will be presented.

Significance of risk programming in forest management

Forest owners are generally uncertain at the time of initial planning about their management conditions. The instability related to management is classified into the risk which can be measured as objective probability, and the uncertainty which can not be measured as objective probability. At the time of initial planning, the risk can be adopted as an indicator for deciding plan, because probability of its estimated value is quantitatively gripped. On the other hand, it is extremely rare that the uncertainty is used as an indicator for deciding plan, because its occurrence is not assured and is subjective. Therefore, in the methods, in which the instability is adopted as a judging criterion in planning the risk alone is usually considered. For this reason, it is called risk programming.

Risk programming has been tried to apply to farm management planning since relatively early date. The major reason why risk programming is required for farm management planning is that yearly changes in weather and climatic conditions strongly affect crop yields, or fluctuation of market price of farm products is greater than that of other products.[6,7,8] In forest management, too, most of the stampage assets, i.e. sources of products, are subjected to natural factors during a long period of time, and forest yield volume to be expected fluctuates greatly by changes of natural factors. In addition, wood price is also very unstable. These items constitute the major risk in forest management.[1,14]

The target of forest management is generally to maximize net return. However, due to the characteristics of forest management mentioned above, it is difficult to predict various inner and outer conditions of management determining the net return, particularly long term price level of inputs and outputs. This makes it extremely difficult to get actual estimation of net return to be expected by adopting a certain plan. However, if the total forest yield volume (hereafter referred to TFYV) in a planned period is taken as the target instead of net return, this approach can be handled more easily, because it excludes price level of inputs and outputs. Maximization of the total forest yield volume results in an increase of net return in most cases, and is not inconsistent with the target to maximize net return.[11,13]

Preliminary consideration on risk programming

When sufficient knowledge on managerial matters is not available, it is considered that the management place their target on maximizing expectation of utility derived from probability distribution of TFYV. Namely, the target is set on the maximization of net utility, i.e., positive utility derived from TFYV minus negative utility caused by variations of TFYV. Accordingly, the utility function held by the management can be called "utility function of forest yield volume with stability preference." As a method to formulate the most appropriate plan which can lead the maximization of utility based on this utility function, some considerations will be given to Heady–Candler model.[2)]

The basic method of this model is not to adopt apparently the abovestated "utility function of forest yield volume with stability preference," but to obtain, by changing continuously the level of $\hat{Z}$ (expected value of TFYV, Z,) which can be estimated as objective function value, locus of combination of $\hat{Z}$ and its variance corresponding to each level of $\hat{Z}$, and locus of working level of each activity, within a limitation of simultaneous linear inequality system. The management can adopt the plan which brings about the maximum utility level as the most appropriate plan, by referring to their own utility function which has intrinsically the offered loci. In this method, it is assumed that the management have utility indifference system as to $\hat{Z}$, and δ_z^2 (variance of TFYV). This relationship can be shown as Fig. 1, in which the order of utility is $U_0<U_1<U_2<U_3$, i.e., the utility increases along the direction to the left-upper portion of the figure.

On the other hand, there is minimum variance attainable for each given value of $\hat{Z}$. In Fig. 2, the area below the curve OP shows combinations of $\hat{Z}$ and δ_z^2 attainable in view of the limiting condition. P corresponds to the highest $\hat{Z}$ attainable. In area above the OP curve, any feasible point does not exist. Namely, OP is the locus of effective point. The Heady–Candler model is to find out such a locus of effective point as above over a whole range satisfying the limiting condition shown by linear inequality. The management select the plan which brings about the highest utility level by referring their own "utility function of yield volume with stability preference" which has the locus of effective point intrinsically. Namely, the point of contact between utility indifference curves in Fig. 1 and locus of effective point in Fig. 2 indicates the most appropriate plan. The Heady–Candler model can be formulated as follows: Under the limiting condition of

$$\sum_{j=1}^{n} a_{ij}x_j \leqq b_i \quad (i=1, 2, \cdots m)$$

$\hat{Z}$ is changed continuously from $\hat{Z}=0$ to max Z, to find out the minimum value of

$$\delta_{z}^2=\sum_{j=1}^{n}\sum_{k=1}^{n}\delta_{jk}x_jx_k.$$

where $\hat{C}_j$: expected value of coefficient of yield volume, C_j

δ_j: variance of C_j

δ_{jk}: covariance of C_j and C_k

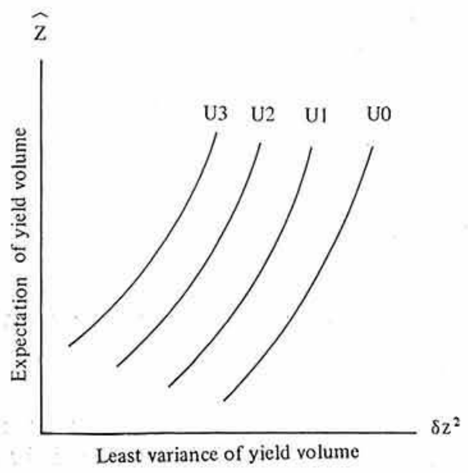

Fig. 1. Indifference curve of forest owner

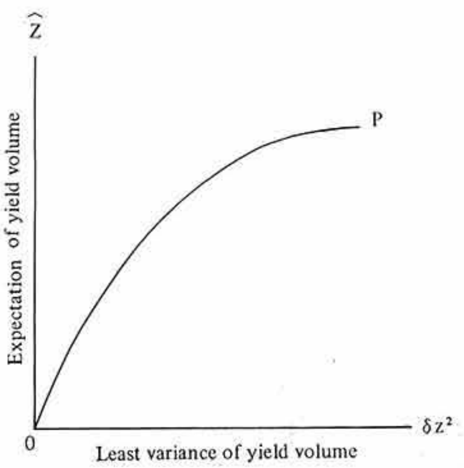

Fig. 2. Locus of effective point

Application to forest management planning

Forest management plan is classified into long-term plan and short term one. The former concerns how to lead the forests to the target, starting from the present state, after making clear what should be the target. Discussions here will be made on the former.

State of forests is generally expressed by elements (state variables) such as area, tree species, forest age, stanpage number, height, volume, growth, etc. The more state variables are used, the closer picture to the actual forest is expressed, but it is impossible to handle a large number of variables in the process of deciding plan by the present mathematical procedures available. However, with the same tree species, even-aged-forest, same site quality and same forest treatment, the state of forests can be expressed by age class area alone. In other words, state variables such as stampage number, height, volume and growth can be determined primarily by a given forest age. Accordingly, any plan to bring forests to the target state within a given scheduled period, starting from the present state, comes to the problem that when and in how much area which one of age classed areas should be cut. This problem is expressed as diagram of forest yield area transition given in Fig. 3.[12]

In this figure, Ai (i=1, 2, ⋯, P) refers to age class area at present state, and Bj (j=1, 2, ⋯, n+p) to age class area of the target forest stand. Also, scheduled period and cutting area are shown as n working period and Xij, respectively. Fig. 3 shows a forest with present age class composition of Ai (i=1, 2, ⋯, p) is led to the forest with age class composition of Bj (j=1, 2, ⋯, n+p) after n working period by repeated planting and cutting. The term Xij expresses that forest stand of i age is cut at the j working period in an area Xij. In this case, replanting is to be made immediately after cutting. On the basis of this diagram of forest yield area transition, conditional formula to induce present forest to the future target are described as follows:

$$A_i - X_{i1} - X_{i+1,2} - \cdots - X_{i+(j-1),j} - X_{i+(n-1),n} = B_{n+i} \quad (1 \leqq i \leqq p)$$

$$X_{1j} + \cdots + X_{jj} + \cdots + X_{p+(j-1),j} - X_{1,j+1} \cdots - X_{n-j,n} = B_{n-j+1}$$

$$X_{1n} + \cdots + X_{nn} + \cdots + X_{p+(n-1),n} = B_1$$

To bring the planning model closer to reality, various limitations are given to activities at each

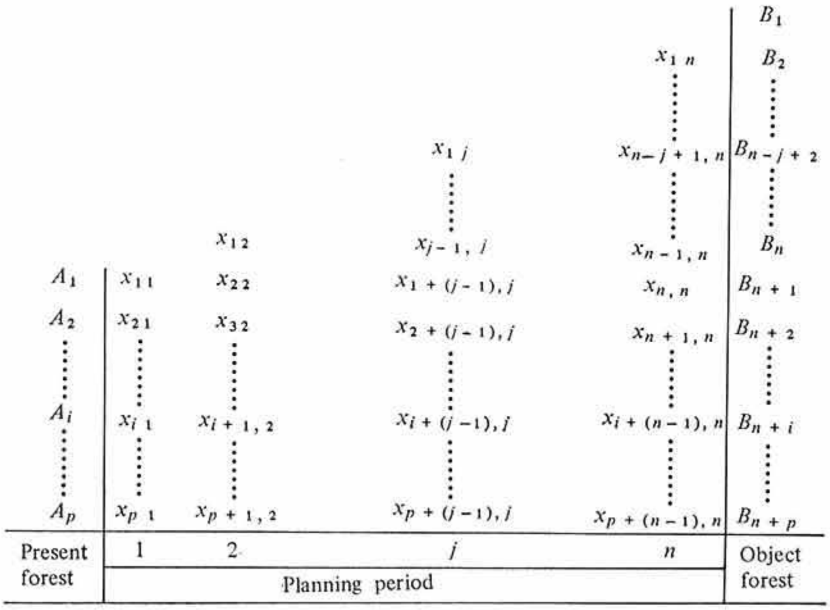

Fig. 3. Diagram of forest yield area transition

							Area (ha)	Age class
							14.39	I
						•	14.39	II
					•	X25	14.39	III
				•	X24	X35	14.39	IV
Age class	Area (ha)		•	X23	X34	X45	14.39	V
I	31.32	•	X22	X33	X44	X55	14.39	VI
II	25.03	X21	X32	X43	X54	X65	14.39	VII
III	26.07	X31	X42	X53	X64	X75	14.39	VIII
IV	47.09	X41	X52	X63	X74	X85	14.39	IX
Present forest		1	2	3	4	5	Object forest	
		Planning period						

Fig. 4. Allotment of variables

working period. Whenever these limitations are related to planting or cutting area by linear proportion, limiting formula fitted to the reality are produced and added to the model. The followings are the limitations at each working period: As related to the yield volume, yield volume at each working period is exactly specified, yield volume at each working period is specified with a certain range, yield volume at a working period is taken as greater than that of the proceeding period, or yield volume at successive period is increased at a certain ratio, etc. As related to input, planting and cutting area is limited to a range fitted to available labor or available wages in each working period, etc. These limitations are adopted singly or in combinations.[9,10]

In this model, Table 1 is taken as age class arrangement of present forest, and normal age class arrangement shown in Table 2 is taken as object forest. The scheduled period is taken as 5 working periods (50 years). Under these basic conditions, the diagram of forest yield area trensition as shown in Fig. 4 is produced. In this planning model, age class unit is 10 years, and young forest of 1–10 years is not cut. To simplify the model, limitations at each working period are all neglected.

Table 1. Present forest

Age class	I	II	III	IV
Forest age	1—10	11—20	21—30	31—40
Area (ha)	31.32	25.03	26.07	47.09

Result and discussions

Actual procedures to solve Heady–Candler model appear in other literatures,[3,4,15] so that the result alone is shown in Table 3. The table is composed of, from the top, plan number, values of each variables, X21—X85, $\hat{Z}$, $\delta_z{}^2$, $\delta_z=\sqrt{\delta_{z^2}}$, $\hat{Z}+2\delta_z$, and $\hat{Z}-2\delta_z$.

In this table, items which are particularly related to the optimum solution are extracted. Least variance corresponding to the $\hat{Z}$, i.e. locus of effective point, is given in Fig. 5. In this figure, which shows plan numbers from 15 to the final step, 78, some of the plan numbers giving the same value of $\hat{Z}$ as the preceeding level are omitted. When $\hat{Z}-2\delta Z$ is called the minimum TFYV possibly attained, and $\hat{Z}+2\delta z$ the maximum TFYV, it is clear from Table 3, that the increase in $\hat{Z}$ is associated with the increase in maximum TFYV, but not always with the

Table 2. Object forest

Age class	I	II	III	IV	V	VI	VII	VIII	IX
Forest age	1—10	11—20	21—30	31—40	41—50	51—60	61—70	71—80	81—90
Area (ha)	14.39	14.39	14.39	14.39	14.39	14.39	14.39	14.39	14.39

Table 3. Activity level of variables

Variable	Plan 54	55	61	67	72	75	78
X21	3.213	2.687	0	0	0	0	0
X31	4.204	4.263	1.144	1.241	0	0	0
X41	6.973	7.445	13.246	14.149	14.390	14.390	14.390
X22	14.196	14.390	12.927	2.953	0	0	0
X32	0	0	0	0	0	0	0
X42	0.194	0	1.463	0	0	0	0
X52	0	0	0	11.437	14.390	14.390	14.390
X23	0	0	0	0	0	0	0
X33	0	0	3.849	5.509	2.424	0	0
X43	3.103	3.159	0	0	0.286	0	0
X53	5.350	5.959	9.073	8.881	11.680	11.680	11.680
X63	5.937	5.272	1.469	0	0	2.710	2.710
X24	0	0	0	0	0	0	0
X34	0	0	0	0	0	0	0
X44	2.734	2.540	0.154	5.851	4.036	3.750	2.540
X54	0	0	4.378	8.539	10.354	10.640	10.640
X64	0	0	0	0	0	0	0
X74	11.656	11.850	9.858	0	0	0	1.210
X25	0	0	0	0	0	0	0
X35	0	0	0	0	0	0	0
X45	0	0	0	0	0	0	0
X55	0	0	0	2.617	10.470	13.180	14.390
X65	4.324	4.799	6.262	2.101	0	0	0
X75	1.932	1.458	8.134	1.558	0	0	0
X85	8.134	8.134	0	8.134	3.958	1.255	0
$\hat{z}$	53.885	53.897	54.045	54.399	54.613	54.643	54.661×10^3 (m^3)
δ_z^2	16.145	16.158	16.530	19.042	21.910	22.712	25.130×10^6
δ_z	4.018	4.020	4.066	4.364	4.681	4.766	5.013×10^3
$\hat{z}+2\delta_z$	61.922	61.937	62.176	63.126	63.975	64.174	64.687×10^3
$\hat{z}-2\delta_z$	45.849	45.858	45.913	45.671	45.251	45.111	44.635×10^3

increase of minimum TFYV. The management can choose the plan which brings about the highest utility level based on the locus of the combination of TFYV and least variance and referring their own "utility function of forest yield volume with stability preference."

According to Fig. 5 it is easily recognized that the vicinity of plan 55 gives optimum solution. The solution of plan 67 coincides with the solution obtained by linear programming. Comparison of it with plan 55 shows that, in spite of the increase of expected TFYV by only 0.764×10^3 (m^3), the variance increased by as much as 8971.371×10^3. From this point, at least, the solution by linear programming is not necessary the most appropriate when variance is taken into consideration. Regarding the values for each variables, plan 78 is the plan to cut a large area of stand with high forest age, while plan 55 includes cutting of stand with low forest age showing the evasion of that risk. Comparison of plan 78 to plan 55 indicates that the latter can attain a higher expected utility, because it contacts the left upper utility function in Fig. 1.

Comparison of application of linear programming to that of risk programming indicates that the former shows a bias to cut the stand of high forest age with higher yield per unit land area. On the other hand, in the latter the target to maximize TFYV is combined with TFYV stability preference, and this dual target is unified by the utility target: degree of preference of TFYV

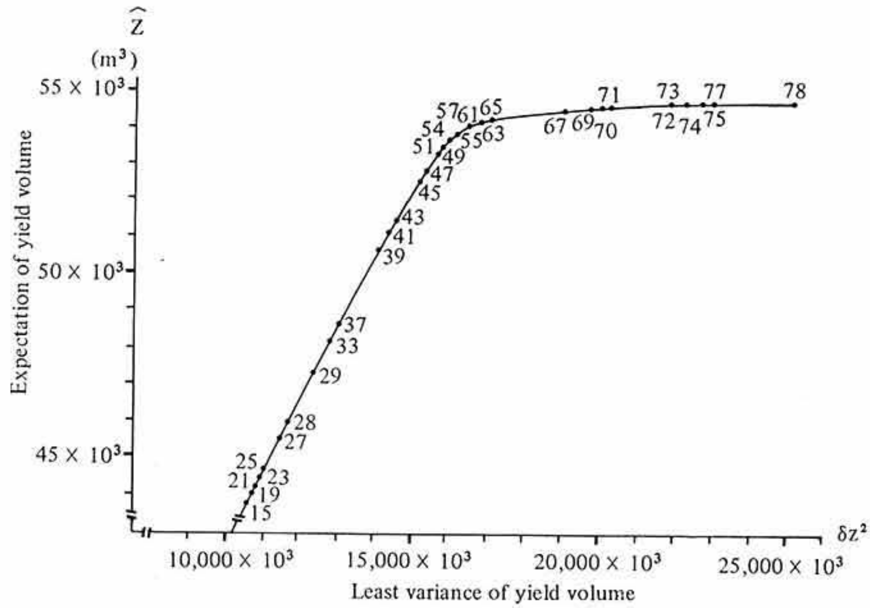

Fig. 5. Locus of effective point (calculated)

stability is determined by the shape of the utility function. As apparently shown in Fig. 5, when the subjective discount rate for the risk becomes greater, a plan which can decrease variation range of TFYV is adopted, although $\hat{Z}$ becomes less. Namely, stand of low forest age is subjected to cutting to evade the risk as a whole.

Forest production is a extremely long term activity lasting for several decades, and at the same time recovery of forest resources is very slow. Therefore, the necessity of sustained yield has been emphasized so far. Unificative handling of yield sustainability, profitability, and stability of profit is an important problem in forest management. From this point of view, a basic concept of forest management planning using risk programming method is presented in this paper.[5]

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