Numerical Estimation of Return Flow in River Basin

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Objective of research

A need for planning proper utilization of water resources has been advocated in recent years. In the agricultural water use also, plans for regional water use and development of procedures for efficient water managements have been taken up as an urgent task. For that purpose, it becomes necessary to grasp the quantity of return flow (water to be used repeatedly). Water for agricultural use is taken from river heads or river water in plains, and most of it is used in paddy fields. The paddy field area has a wide extent and hence shows a great evapotranspiration, but it conserves groundwater, causes continuous return flow through field surface storage of water, and therefore plays an important role in the repeated utilization of water resources, by supplying water to river-maintaining flow or to adjacent areas. This return (repeated) flow is a pronounced characteristic of agricultural water use, and is unavoidable in planning regional water use, in view of the recently concerned shortage of water resources. A method for numerical estimation of return flow has to be developed, as an urgent task.

A research by the authors is now in progress to analyse regional water balance in water system, including the estimation of return flow, by adopting a method of system analysis, which employs multiple regression analysis, a method of multivariate analysis, to solve the integral equation shown by Wiener applied to the problem of runoff mechanism in mountains paddy fields, that is inevitable in water management in river basins. In this paper, a method for numerical estimation of changes by time and locational distribution of paddy field return flow by the use of a multiple regressional model will be presented together with an example of its practical application.

Method of analysis

1) Basic formula

Taking a river basin as a black box, and if the rainfall-runoff relation is supposed to follow the Wiener's nonlinear system theory, the relation can be expressed generally by the following Volterra series, taking the factor t as the present time, i.e. starting time:

$$y(t)=\sum_{n=0}^{\infty}\int_{(n)}\cdots\cdots\int_{0}^{\infty}h_n(\tau_1,\tau_2,\cdots\cdots,\tau_n)\prod_{k=1}^{n}X(t-\tau_k)\,d\tau_1,d\tau_2,\cdots\cdots,d\tau_n \quad\cdots\cdots\cdots\cdots(1)$$

where

$X(t\text{-}\tau_k)=Ar\cdot R(t\text{-}\tau_k)$, Ar: Catchment area, $R(t\text{-}\tau_k)$: rainfall intensity, $y(t)$: the value of the watershed runoff at time t, h_n: runoff kernel of the n th degree, τ_k : the variable showing time-lag.

In the present paper, the equation (1) is approximated practically up to the term of the second degree of moment, and multiple regression analysis, one of the discrete expression, is employed to solve the approximated equation. Then, by expressing the best unbiased estimation of partial regression coefficients as A_0, A_i, and B_{ij}, and an estimated value of runoff as $Y(t)$, the following equation is obtained:

$$Y(t)=A_0+\sum_{i=1}^{n} A_i \cdot X(t-\tau_i)$$
$$+\sum_{i=1}^{m}\sum_{j=i}^{m} B_{ij}\cdot X(t-\tau_i)\cdot X(t-\tau_j) \quad ...(2)$$

The parameters as A_0, A_i, and B_{ij} are determined by the method of least squares. Here, if the normal equation, as it is, is solved to obtain the coefficients for linear terms A_i and ones for nonlinear terms B_{ij} simultaneously, an independency between the above two kinds of predictor variables can not be kept by reason of the presence of a high correlation between them, because multicolinearity is created in the solving process of the normal equation, and it makes the partial regression coefficients obtained less reliable. Therefore, an attempt is made to remove a linear component from the observed values of $Y(t)$ and apply a linear multiple regressional model, which is expressed by only the linear terms A_i to the linear component, while the nonlinear component which can not be expressed by the linear multiple regressional model, i.e. a residual difference between observed value and estimated value by the linear model is treated separately by a nonlinear multiple regressional model, which is represented by nonlinear terms B_{ij}.

2) Runoff model for paddy field area

The above equation (2) can be regarded as a general model concerning rainfall-runoff, and is used mainly to analyze the runoff in mountains. However, to apply this equation to river basins including paddy field areas, and further to incorporate a cycling process of repeated water use, the following processes of water in-coming and out-going must be included in the model:

(1) A component flowing down along rivers directly from mountains.

(2) A component flowing out through drainage canals. It comes from rain water in a given area, which is stored temporarily on paddy fields, and subsequently flow out by percolation or through broken parts of levees of paddy fields.

(3) A component, which is once intaken in an area for irrigation, spread over the area, and returned to rivers together with runoff of rain water.

(4) A component coming into an area from adjacent, outside areas.

(5) A component flowing out to outside areas.

A model to which the above in-coming and out-going components are incorporated is expressed as follows:

$$Y(t)=F(t)+A_0+\sum_{i=1}^{n} A_i\cdot Z_i$$
$$+\sum_{i=1}^{m}\sum_{j=i}^{m} B_{ij}\cdot Z_i\cdot Z_j \quad ......(3)$$

where $F(t)=\sum_{j=1}^{d}(Q(t)_j-\sum_{k=1}^{l} q(t)_{kj})$,

$Z_i=X(t-\tau_i)+q(t-\tau_i)+q'(t-\tau_i)-q''(t-\tau_i)$,

$X(t-\tau_i)=A_r\cdot R(t-\tau_i)$,

$\tau_i=(i-1)\cdot \Delta t$

(Z_j, $X(t-\tau_j)$, τ_j, etc are similar to the case of i), and

$Y(t)$: estimated flow at a fixed measuring station in paddy field area

$Q(t)_j$: flow in tributaries

$q(t)_{kj}$: intaken water from rivers in the area

A_0: regression constant

A_i: partial regression coefficient of linear runoff

B_{ij}: partial regression coefficient of nonlinear runoff

$R(t\text{-}\tau_i)$; $R(t\ \tau_j)$: rainfall in the area

$q(t\text{-}\tau_i)$; $q(t\text{-}\tau_j)$: total quantity of water intaken in the area

$q'(t\text{-}\tau_i)$; $q'(t\text{-}\tau_j)$: irrigation water supplemented from outside areas

$q''(t\text{-}\tau_i)$; $q''(t\text{-}\tau_j)$: a component which flowed out to outside areas

Ar: acreage of the basin

Δt: time increment

t: time

The $F(t)$ of the equation (3) is a function expressing water balance system in which

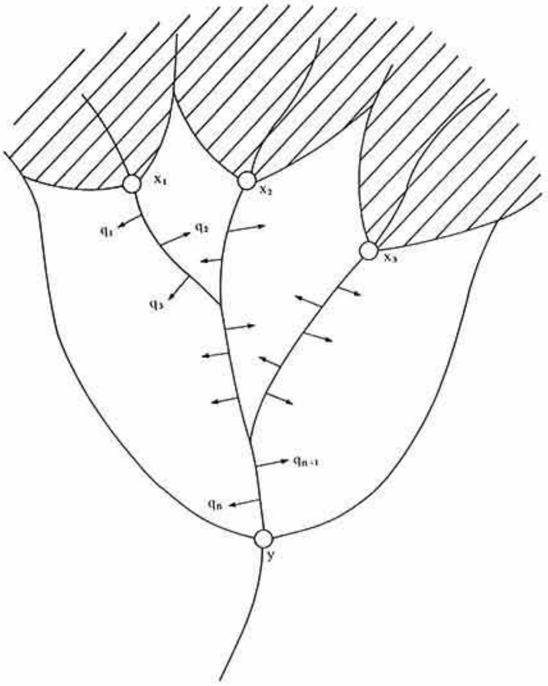

Fig. 1. Outline of a river basin

repeated use of irrigation water is taken into consideration, and it indicates residual flow of rivers. Namely, in a river basin as shown in Fig. 1, $x_1 \sim x_3$ represent flows at fixed measuring station in the mountain area, y represents a flow at a fixed measuring station in the paddy field area, and $q_1 \sim q_n$ are regarded as quantity of water intaken at inlets in the paddy field area.

A hatched portion represents mountain area of the basin, and the portion between $x_1 \sim x_3$ and y is regarded a paddy field area of the basin. If it is postulated that the size of the basin is the one in which the river flow reaches to a fixed measuring station in the paddy field area from the fixed measuring stations in the mountain area with a time less than the time unit for estimation, the water balance system in this river basin can be considered as follows:

The river water decreases in quantity by water intakes during its flowing down in the paddy field area, and at a certain point the intake may exceed river flow. In this case, the short is made up by return flow. This relation is expressed as follows:

$$\left.\begin{array}{l}\text{when } x \geq q,\ q'=q,\ x'=x-q \\ \text{when } x<q,\ q'=x,\ x'=0\end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots(4)$$

where

- x: River flow,
- q: Water quantity for water right,
- x': Residual river flow, and
- q': Possible water intake (water actually intaken)

By this method, the residual river flow, $F(t)$ at each point up to y, can be obtained by subtracting $q_1 \sim q_n$ from respective $x_1 \sim x_3$. When the equation (3) is transformed to $Q_\epsilon(t)=Y(t)-F(t)$, $Q_\epsilon(t)$ represents residual flow at the fixed measuring station. Accordingly, the equation (3) becomes to

$$Q_\epsilon(t)=A_0+\sum_{i=1}^{n} A_i \cdot Z_i+\sum_{i=1}^{m}\sum_{j=i}^{m} B_{ij} \cdot Z_i \cdot Z_j \quad \ldots(5)$$

and $Q_\epsilon(t)$ is considered to be caused by rain water runoff and return flow of intaken water in a given area, so that the equation (5) offers a solution of the multiple regressional model of the equation (2). In actual runoff, the intaken irrigation water is considered to flow out combined with rain water. In a sense, the intaken water is regarded as rain water artificially spread over paddy fields. Accordingly, the components of water intake and incoming or out-going water from or to outside areas in the equation (3) and (5) are included into the term of rainfall, and expressed as Z_i and Z_j. Furthermore, in the equation (5), A_i expresses hydrologically a statistical unit hydrograph, and the sum of partial regression coefficient, $\sum_{i=1}^{n} A_i$, expresses runoff coefficient of linear runoff. As the water intake is considered as an artificial rain as stated above, the time lag of the runoff is regarded as the same, $\sum_{i=1}^{n} A_i$ is also considered as a reduction ratio for water intake components. The reduction ratio, referred here, means a ratio of out-going water (D_2) to the outside areas as surface water to the intaken water (D_1) in a given area, i.e. $r=\frac{D_2}{D_1}$. When the model is

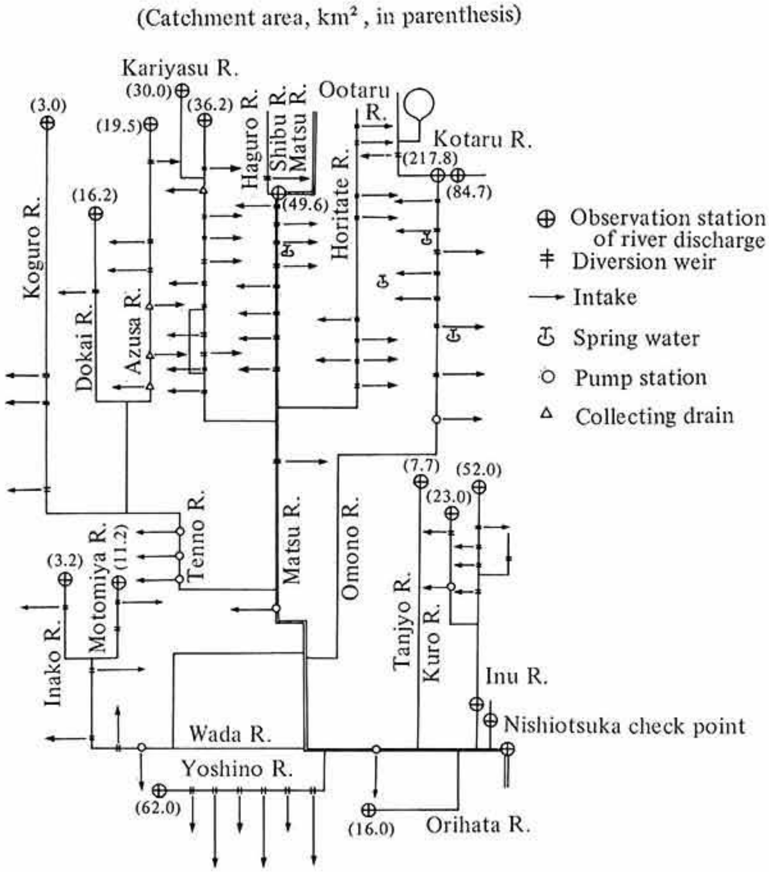

Fig. 2. Present situation of irrigation system in the Yonezawa Plain project area

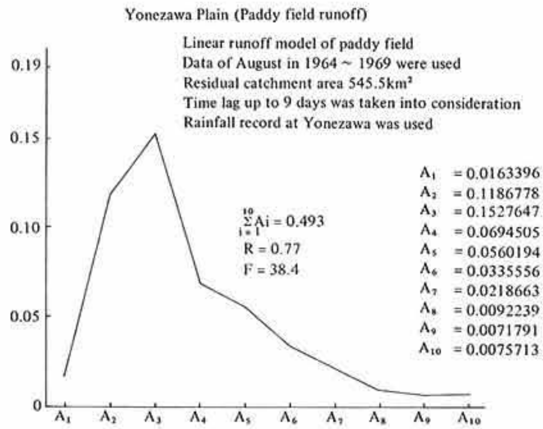

Fig. 3. Characteristics of paddy field runoff (at Nishiootsuka in the Yonezawa Plain project area)

determined, the return flow is estimated as follows: Assuming that there is no rain in a given area during the irrigation period, $R=0$, and water intakes, water supplemented from outside areas and water flowed out to outside areas for q, q' and q'', respectively, are introduced into the model of equation (5). Namely, the return flow at every moment is estimated in the form of neglecting rain component. In this case, a regression constant $A_0=0$.

Result of analysis

Fig. 2 shows the present irrigation water system in Yonezawa Plain project area, located along the uppermost stream of Mogami River.

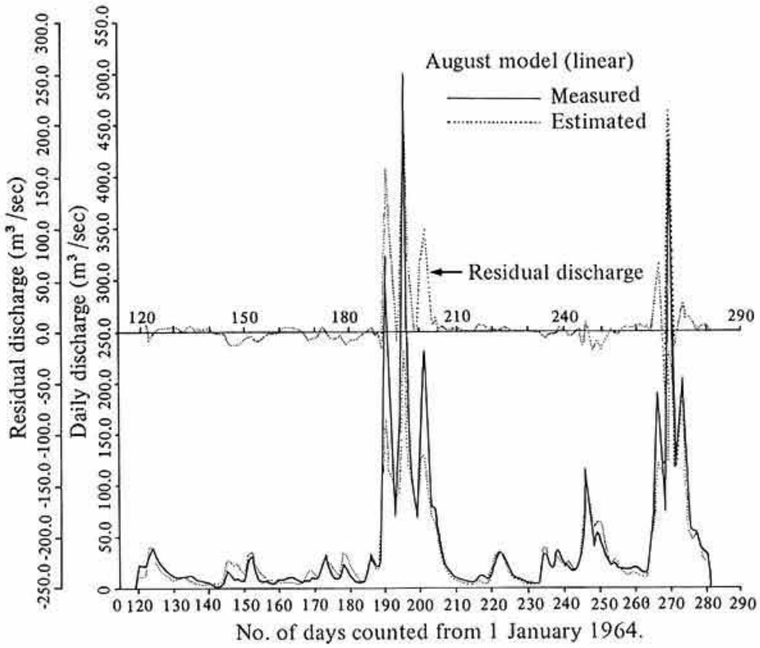

Fig. 4. Result of estimation of paddy field runoff (in 1964 at Nishiootsuka in the Yonezawa Plain project area

According to this water system, water balance calculation stated with the equation (4) was carried out using data actually measured. From this calculation, $F(t)$ shown in the equation (3) was obtained, and the multiple regressional model of the equation (5) was estimated. As the optimum model for paddy field runoff at Nishiotsuka, a fixed point of the Yonezawa plain, an estimation from the data of August in 4 years was adopted. Fig. 3 shows a statistical unit hydrograph, illustrating linear runoff characteristics, from which the ratio of return flow at Nishiotsuka is obtained as 49.3%.

Fig. 4 shows an example of fitness of the paddy field runoff model at Nishiotsuka. This model was obtained from data of August, but in other months the actual values and estimated values coincide each other very well. Furthermore, it was recognized that the linear runoff model alone is good enough for the estimation. From this, it is considered that the runoff of the paddy field area is predominantly linear, and the peak floods at the fixed measuring station in paddy field area are mostly occupied by the component which flowed down along the rivers from the mountains, rather than runoff component within the paddy field area.

Fig. 5 shows predicted values of return flow at Nishiotsuka, a fixed station for measurement, and Fig. 6 indicates a plan for improving the current irrigation system. Fig. 7 gives a predicted runoff at a point immediately upper from Nishiotsuka at the time when the improvement project for the present irrigation system is completed. For this prediction, rainfalls in 1968 as a standard year and the planned water intakes in the project area shown in Fig. 4 were introduced into the paddy field runoff model given in Fig, 3. From Fig. 7, it can be understood that the rate of flow after the completion of the project exceeds the actually measured rate of flow at Nishiotsuka, particularly in a droughty period the return flow originated from supplemented water from a dam constitutes a substantial part.

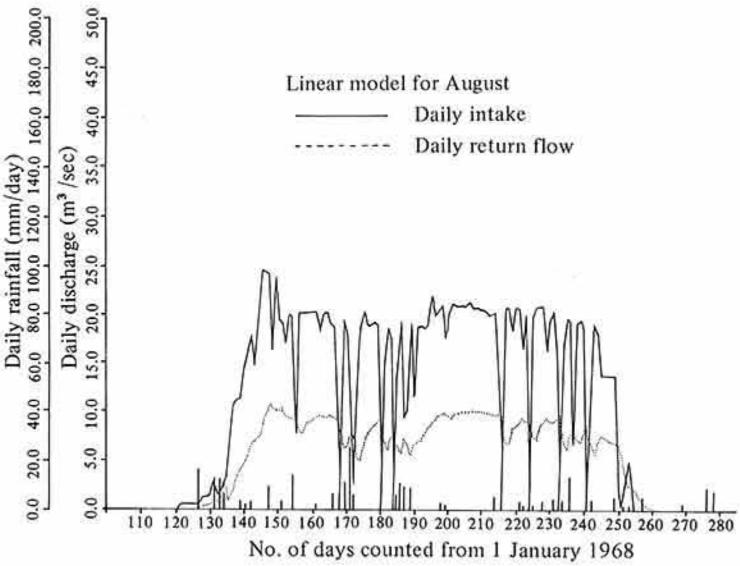

Fig. 5. Estimated daily rate of return flow, at Nishiootsuka of Mogami River in 1968 (a standard year).

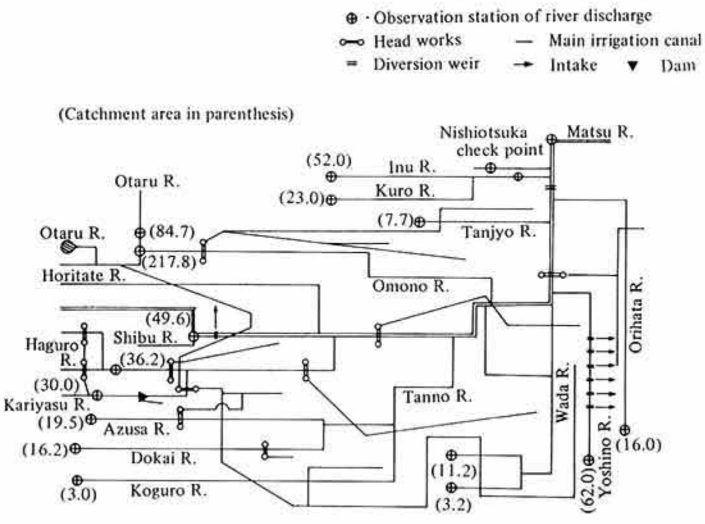

Fig. 6. Planned design of irrigation system (Yonezawa Plain project area)

Conclusion

As stated above, the multiple regressional model expressed by the equation (3) is the model in which the water recycling, especially the repeating use of water, is taken into consideration. Though statistical method is employed for its analysis, it has a characteristic of grasping the phenomena by average values. Furthermore, it divides a river basin into blocks, and by connecting runoff models of

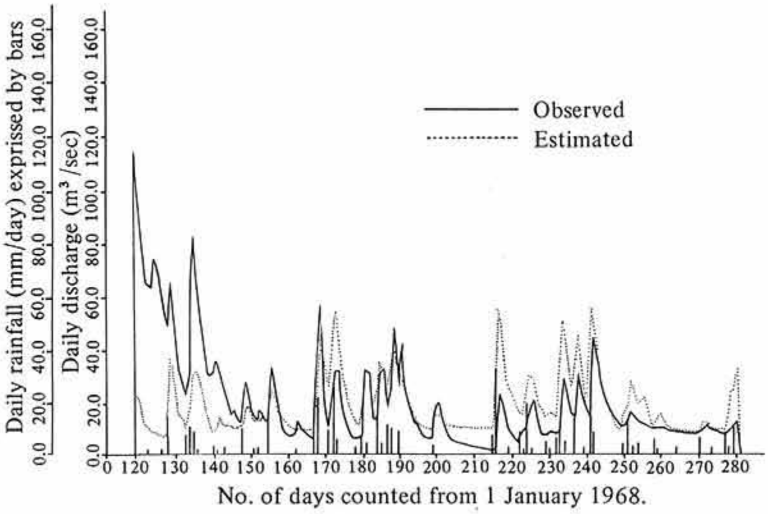

Fig. 7. Predicted runoff after the completion of the project. 1968 was taken as a standard year

respective blocks, it appears to be a significant model from a viewpoint of physics or as a regional water balance system. The great characteristic of agricultural water use is that the water consumption is small as represented by evapotranspiration and the residual water is utilized as a new water source for adjacent down-stream areas. The water use including repeating use can be defined as "low-water management", which utilizes efficiently the natural water recycling function. Based on the numerical estimation of repeating use of water, that makes it possible to expect how much is it by time and locations, a highly efficient water management can be practiced by an appropriate application of the recycling system of agricultural water use.

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