A Method of Developing Stumpage Volume Tables and Stand Yield Tables

By HITOSHI AWAYA

Forest Management Division, Forestry and Forest Products Research Institute

Stumpage volume table

Stumpage volume tables presently being used for national forests in Japan have been developed for areas under the jurisdiction of every regional forest offices, by an uniform method based on the "Working rule for preparing stumpage volume tables for important tree species," which was established by the Forestry Agency in 1955. The program of developing stumpage volume tables completed its first stage in 1966 with the production of 68 kinds of tables. These tables are used in most private forests also, but some prefectures like Miyagi and Fukushima produced their own tables for private forests based on the "Working rule."

The stumpage volume tables in Japan are to determine the total stem volume outside bark (in m^3) by measuring diameter breast high and total tree height, and the stumpage volume is expressed by every 2 cm of diameter breast high and every 1 m of tree height.

To produce a table, sample trees are selected and cut down in the stand of a given area. Using the Hüber's formula, the volume is calculated by sectional measurements with an interval of 2 m distance from ground level. The volume of tree tops shorter than 2 m is determined by taking them as cones. The sum is the total stem volume of sample trees. Number of sample trees is mostly 400-1000.

The data thus obtained is divided by diameter class by 10 cm, and coefficients of the following tree volume formula are determined by the least-square method:

$$\log v = \log b_0 + b_1 \log d + b_2 \log h \quad \cdots\cdots (1)$$

where, v: total stem volume, d: diameter breast high, h: total tree height, and b_0, b_1, b_2: coefficients

Taking $Y = \log v$, $X_1 = \log d$ and $X_2 = \log h$, and expressing diameter class by subscript i, the above formula can be rewritten as follows:

$$Y_i = \log b_{0i} + b_{1i} X_{1i} + b_{2i} X_{2i} \quad \cdots\cdots (2)$$

Residual mean square of the equation (2), calculated for each diameter class, was subjected to the chi-square test to examine the homogeneity of variance, and the group of diameter classes which show chi-square values, calculated by the equation (3), not exceeding at a 0.01 probability level the values in chi-square table at a degree of freedom (number of 10 cm diameter class-1) was selected.

$$\left.\begin{aligned} M &= (2.3026\,[(\Sigma f_i)\log \bar{s}^2 - \Sigma f_i \log s_i^2]) \\ C &= 1 + \frac{1}{3(a-1)}[\Sigma \frac{1}{f_i} - \frac{1}{\Sigma f_i}] \\ \chi^2 &= M/C \\ \bar{s}^2 &= \Sigma f_i s_i^2 / \Sigma f_i \end{aligned}\right\} \cdots\cdots (3)$$

where, f_i : number of sample trees in a diameter class,

s_i^2 : residual mean square of a diameter class

a : number of diameter classes

With consecutive diameter classes showing no significant differences in residual mean square, analysis of covariance was made as given in Table 1.

Coefficients determined with the lump of data of diameter classes showing F values not exceeding F-table values at a 0.05 probability level are used in tree volume formula for combined diameter classes. As the equation (1)

Table 1. Analysis of covariance

Factor	Degree of freedom	Sum of squares	
Residual sum of squares	$\sum^{a}(f_i-3)$	$\sum^{a}(\sum y_i^2-b_{1i}\sum x_{1i}yi-b_{2i}\sum x_{2i}y_i)$	Ⓐ
Common regression	$(\sum^{a}(f_i-1))-2$	$\sum^{a}\sum y_i^2-b'_{1i}\sum^{a}\sum x_{1i}y_i-b'_{2i}\sum^{a}\sum x_{2i}y$	Ⓑ
Difference for slopes	$2(a-1)$	Ⓑ−Ⓐ	Ⓒ
Total regression	$\sum^{n} f_i-3$	$\sum^{n} y_i^2-b''_{1i}\sum^{n} x_{1i}y_i-b''_{2i}\sum^{n} x_{2i}y_i$	Ⓓ
Difference for levels	$a-1$	Ⓓ−Ⓑ	Ⓔ

Test of difference in slopes : F =〔Ⓒ/ 2 (a − 1)〕/〔(Ⓐ/$\sum_{a}$(fi− 3)〕

Test of levels : F = 〔Ⓔ/(a − 1)/Ⓒ〕/〔 2 (a − 1)〕

is logarithmic, the estimated volume tends to be smaller, so that the values to be adopted to the stumpage volume table were calculated by multiplying correction factor to the estimates obtained by the equation (1). And also, the gap at the boundary of diameter class was corrected by the moving average techniques.

$$C=10\ \exp\left[\frac{f-1}{f}\left(\frac{1}{2}\log e\ 10\right)s^2\right] \quad \cdots\cdots(4)$$

where, f : number of sample trees, s^2: residual mean squares.

The stumpage volume tables were produced by each regional forest office with the guidance given by the Forest Experiment Station, but the tables for fir and spruce growing in sub-alpine zone of Kanto and Chubu-regions were formulated by the Forest Experiment Station.[4]

Stand yield table

Prior to the development of stumpage volume tables, it was decided in 1949 to develop stand yield tables according to the "Working rule for preparing stand yield tables for even-aged pure stands," but it took 20 years to develop 36 kinds of stand yield tables, because the program of stand yield tables and that of stumpage volume tables were carried out simultaneously.

Sample plots to be used for preparing stand yield tables were selected in areas with the following conditions:

1) Even-aged pure stand showing healthy growth without any damage.
2) No abnormal space in stand canopy.

Size of sample plot is 0.1—0.2 ha. With all stand trees in the sample plots, diameter breast high and total tree height are measured, tree volume was estimated by using stumpage volume table, and the trees are grouped into thinning trees and survival trees by the Terazaki-B type thinning method. With the measurements of the survival trees thus obtained in the sample plots, relationships among stand components shown below and the distribution range of measurements are examined. Sample plots beyond this range are rejected.

1) Stand age *vs* number of trees per ha.
2) Stand age *vs* basal area per ha.
3) Stand age *vs* stand volume per ha.
4) Stand age *vs* mean diameter.
5) Mean diameter *vs* number of trees per ha.

To determine these relationships among stand components various curved formulae are used. The yield table for Hinoki in Kanto region,[3] developed by the Forest Experiment Station, employed the following equations:

$$T\ vs\ N \text{ and } \overline{D}\ vs\ N : N=ae^{bx} \quad \cdots\cdots(5)$$

$$\text{For others : } Y=a\ T^{5}\ e^{-\frac{C}{T}} \quad \cdots\cdots(6)$$

where, T: stand age, N: number of trees, $\overline{D}$: mean diameter, Y: other stand-component factors

Other curved formulae as given below are also used in many cases.

$$Y=\frac{T^2}{a+bT+cT^2} \quad \cdots\cdots (7)$$

$$Y=10 \exp \left(\frac{T}{a+bT}\right) \quad \cdots\cdots (8)$$

$$Y=a\,x^b \quad \cdots\cdots (9)$$

Next, including the relation of stand age *vs* mean tree height, standard deviation of measurements around the guide curve obtained by the equation (5) or (6) was calculated for each age class, and corrected standard deviation was determined by drawing curves by free hand. At the upper and lower sides of the guide curve, an area with a width of 1.5–2.0 times corrected standard deviation was taken, and this area was divided equally into 3 sections. From the uppermost section, they were referred to the first, second and the third site classes. Sample plots which showed different site class in the similar site classification with each factor were mostly rejected. In such examination, 38 out of 150 sample plots were rejected in case of the above-mentioned yield table.

Taking the values shown by the guide curve of stand age *vs* mean height as the mean height of the second site class, middle values of each section were taken as mean height of each site class. Using the measurements of sample plots which were included in each site class classified by this relationship, curved formulae showing the following relations among the stand components were obtained for each site class, in addition to the above-mentioned relationships:

1) Mean height *vs* mean diameter.
2) Mean diameter *vs* basal area per ha.
3) Mean height *vs* stand volume per ha.
4) Basal area per ha *vs* stand volume per ha.

Curved formulae applied are the same as above.

By mutual checking of these relationships, values of stand construction factors against the stand age of survival trees are determined at an interval of 5 years.

Components of thinning trees are determined by the following method.

1) Diameter breast high: Values estimated by a linear regression equation between mean diameter of survival trees and that of thinning trees, and values estimated by a linear regression between ratio of all trees to thinning trees in number and the ratio in sum of diameter are mutually checked.
2) Number of trees: Number of survival trees at each age class is subtracted from the number of trees at a preceding age class.
3) Tree volume: Relation of stand age *vs* stand volume for all trees is drawn by free hand, and value of stand volume for survival trees is subtracted from value of stand volume for all trees. Several other methods such as the use of a linear regression between ratio in tree number and that in stand volume, etc are also available.

As the stand yield table is produced by the mutual checking of stand age and stand components as well as analytical study with 2 variables among factors, ample experiences on growing process of forests are needed.

In recent years, studies are in progress on the method of estimating directly stand volume per ha from mean height and number of trees per ha by using reciprocal equation of yield-density effect,[2] and stand volume equation.[1] In this method, the mean height is to be estimated from height curves by site class, described above, or from growing process obtained by site index curve, and the number of trees is to be estimated by using the natural decreasing pattern.

$$V=(b_1\,h^{b_2}+b_3\,h^{b_4}/N)^{-1} \quad \cdots\cdots (10)$$

$$\log V=b_0+b_1 \log h+b_2 \log G$$
$$\log \bar{d}g=b'_0+b'_1 \log h + b'_2 \log N \quad \cdots\cdots (11)$$
$$G=\frac{\pi}{4}\cdot \bar{d}g^2 \cdot N/10^4$$

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even-aged pure stand. *Bull. Gov't. Forest Exp. Sta.*, **265**, 102 (1974) [In Japanese with English summary].

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