## Theoretical Approach to the Hand Tractor of Rotary Tillage

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In the previous JARQ Vol. 8, No. 3, the author presented basic theories for the whole machine motion of the rotary power tiller and its design principle under the title of "Machine Stability of Rotary Tillage".

In this issue, digests of motion analysis which provide for a better design of knives are introduced. This paper also includes some material published in a book in 1962<sup>2</sup> and other research<sup>3</sup> undertaken by the author after the 1962 publication of the book.

## Motion analysis and design theories of rotary knives

#### 1) Locus curve equation of a rotary knife motion

(1) Types of tillage hoes

Although there are many types of ratory hoes in the world, the two typical types in Japan are the pick-tine and knife type as shown in Fig. 1.

(a) of Fig. 1 shows the "pick-tine" type that is called "Futsu-zume" in Japanese, while

the (b) illustration is called the "rotary knife" or "rotary blade", and is called "Nata-zume" or "Nata-ba" in Japanese.

Smaller tillage resistance is expected, in general, from the pick-tine type than the rotary knife. However, in Japan on paddy rice fields covered with grass and straw, picktine is easily clogged with grass and straw. The rotary shaft from which the times project soon becomes full of coiled grass and the drum effect produces inefficient tillage and increased resistance.

In Japan, the rotary knife type is much more popular than the pick-tine. The picktine type is most effective for cultivating upland fields which have little grass. The rotary knife type is discussed in the first part of this report.

(2) Locus curves for rotation of rotary knives

When the center of the rotary axle is stationary and the rotary blades are rotating, the radius of rotary blades describes a circle. As the center of the rotary axle moves hori-



Fig. 1. Two typical types of hoes in Japan



Fig. 2. Locus curve of rotary blade motion

zontally or nearly so, the locus curve of the radius of rotary blades becomes a "trochoid curve". See Fig. 2.

Following are the symbols:

 $2 - b^{\prime}$ 

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ω=the angular velocity of a rotary blade, radians/second, where

$$\omega = \frac{n\pi}{30} \tag{1}$$

- v=the velocity of travel of the center of the rotary axle, cm/second.
- a=the radius of an imaginative co-axial circle which is rolling on x'-axis as shown in Fig. 2, cm.

$$a = \frac{v}{\omega}$$
 (2)

R=radius of rotary blade, cm.

- $\theta$ =rotation angle, radians
  - n=revolution of rotary shaft, r.p.m.

H=depth of cut or tillage, cm.

The trochoid curve in Fig. 2 has the following characteristics:

The velocity of travel, v, is directly proportional to the imaginative radius, a, of the co-axial circle.<sup>1/B</sup>

The angular velocity,  $\omega$ , of the rotary blade is inversely proportional to the imaginative radius, a, of the co-axial circle.

A portion of the radius R, of the rotor is non-tilling, thus, the depth H of tillage is always less than radius; R, of the rotary blade. The locus of the trochoid curve shown in Fig. 2 is indicated by

$$\begin{cases} X = vt - R \sin \omega t \\ Y = R - R \cos \omega t \end{cases}$$
(3)

where,

t = time as parameter, second.

The gradient of the tangential line to the locus curve of the rotary blade is,

$$\frac{dy}{dx} = \frac{(\frac{dy}{dt})}{(\frac{dx}{dt})} = \frac{R\omega \sin \omega t}{v - R\omega \cos \omega t} \quad (4)$$

When the rotary blades are rotating, there are several calculations that may be obtained from X, Y and t of Fig. 2 and Equation (3). Some examples are given as follows:

(1) Velocity of motion, V, to rotary blades

$$V = ds/dt = \sqrt{(dx/dt)^2 + (dy/dt)^2}$$

(2) The direction of motion,  $\tan \tau$ 

$$\tan \tau = dy/dx = \frac{(dy/dt)}{(dx/dt)}$$

(3) Actual values and effects of their rotating motions

(i)  $\theta_1$ : The angle between the axle centerhorizontal line and the tangential line of the locus curve at the point of intercept is shown in Fig. 3. The tip of the rotor blade does not cross the horizontal O-line vertically, but at some specific angle, which is less than 90°.



The estimate of  $\theta_1$  is expressed by

$$\theta_1 = \tan^{-1} \left( \frac{-Rn\pi}{30v} \right) \tag{5}$$

here, the letter-factors are as defined in Section b.

For a sample assumption of R equal to 23 cm, n equal to 150 r.p.m., and v equal to 60 cm/second, the value for angle  $\theta_1$  is approximately 84°34'. The general estimates or general calculated values of  $\theta_1$  is from 80° to 88°.

(ii) a: Fig. 3 also shows the point at which the tangent to the trochoid curve becomes vertical. The tangent of the locus curve becomes vertical at "a cm" below the honrizontal O-line after the rotor blade crosses the horizontal O-line.

At the rotor blade position shown in Fig. 3, the differential value of dy/dx is equal to infinity  $(dy/dx=\infty)$ . The vertical distance "a cm" from the horizontal O-line to another horizontal line through the point where the tangential line of the locus curve becomes vertical is also the radius of the imaginative co-axial circle, expressed by

$$a = \frac{v}{\omega} = \frac{30v}{n\pi} \tag{6}$$

The smaller the velocity v, the smaller the

radius of the imaginative co-axial circle becomes, and the greater the value of angle  $\theta_1$ is at the crossing point of the horizontal O-line and the locus curve.

Generally,  $a=1\sim4$  cm

(iii) l: The horizontal distance between the point of intercept on the O-line by the locus curve and the point when the tangential line becomes vertical is estimated by,

$$l = \frac{15v \left\{ \pi - 2\cos^{-1}\left(\frac{30v}{Rn\pi}\right) \right\}}{n\pi} \quad (7)$$

*l* is generally very small.

(iv)  $H_{\text{max}}$ : The maximum depth of rotary tillage which Fig. 4 shows that the bottom of the reduction case, in general, controls the maximum depth of the rotary tiller cut. When the bottom of the reduction case contacts the field untilled soil surface, it limits the depth of tillage or cut.

Therefore, the maximum depth of cut for common tillage is,

$$H_{\max} = R - a_1 \tag{8}$$

Thus, R consists of  $H_{\max}$  and  $a_1$ , here,

#### $a_1$ =impossible tilling radius

For the Japanese power tiller, the general value of R ranges from 20 to 25 cm, and  $H_{\text{max}}$  is approximately 13 to 18 cm with actual values of  $a_1$  approximately equal to 5~7 cm.



Fig. 4. Relation between the bottom of reduction case and tilling depth

And  $a_1$  is presumed as the radius of the semi-circle bottom portion of the reduction case.

## Important design factors relating to the performance of rotary blades and their shapes

#### 1) Main factors

The tillage performance of rotary baldes consists of three dominantly important factors. These are:

Rake angle,  $\beta_1$ , (Hai-Kaku in Japanese as named by the author), is one of the factors that affect the clod throwing action. The other factors are the speed of the blade and the total machine velocity. The edge curve of the blade influences grass and straw entwinement. The edge shape is also important.

The shape of blade sections influences the torque-characteristics of the blade.

(1) Rake angle,  $\beta_1$ 

Fig. 5 shows the actual location of the rotary blade rake angle  $\beta_1$  as the angle between the rotor radius direction and the tangential line of tip-outside surface of the blade when it intersects the locus curve. The angle between the tangential line of tip-outside surface of the blade and the tangential line of tip locus curve is the relief angle,  $\gamma$ .

 $\beta$  is the angle between the radius direction from origin O to the tip of the rotary knife edge and the tangential line of the locus



Fig. 5. Rake angle and relief angle

curve. Therefore,

$$\beta_1 = \beta - \gamma \qquad (9)$$

where  $\beta$  is estimated analytically by

$$\beta = \cos^{-1} \left\{ \frac{30v}{R} \sqrt{\frac{H(2R - H)}{(30v)^2 - 60n\pi v(R - H) + (Rn\pi)^2}} \right\}$$
(10)

Then, when the maximum depth of cut  $H_{\text{max}}$  is substituted for H of equation (2-10), the  $\beta$  value obtained becomes the special angle at the time when the tip of blade crosses the soil surface, as shown in Fig. 5.

When H is maximum, the general value for  $\beta$ ,

$$\beta_{\rm max} = 80^{\circ} \sim 87^{\circ}$$

Relief angle,  $\gamma$ , may have the following characteristics:

(1) The least resistance is when  $\gamma$  has a value less than 20° but greater than 5°.

(2) Medium resistance when  $\gamma$  has a value of around 20°.

1.4

(3) High resistance results but there is

a good effect on soil-throwing action of rotary blade when  $\gamma$  is greater than 20° up to 40°.

As a result, the  $\beta_1$  value can be conceptionally stated as follows:

- 40°~55°—For soft soils such as, sandy or muddy soil.
- (2) 55°~75°—For normal soils such as sandy loam, loam, or clayey loam soil.
- (3) 75°~85°—For hard soils such as heavy clay or dry soil.

#### 2) Edge curve of the blade

The shape of the edge curve has some influences on the entwining of grass or straw around the blades and exle, and the tilling resistance is caused by the friction between the side face of knife body and the soil. Fig. 6, shows the location of the edge curve on a blade, the frictional area of the blade with the soil and the tangential line from the edge curve of the blade.

The concepts for angle  $\phi_1$  between the

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- C Y



Fig. 6. Edge curve of the blade



Fig. 7.

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(1) Less entwining of grass or straw occurs when  $\phi_1$  is more then 57° and greater entwining of grass or straw occurs when  $\phi_1$  is less than 57°<sup>1)</sup>.

(2) However, the greater  $\phi_1$  forms larger frictional areas of the knife body with the soil, and tilling frictional resistance thus becomes large. The smaller  $\phi_1$ , conversely forms smaller frictional area of the blade with the soil and tilling resistance is thus smaller.

Therefore, on a field where there is no grass or straw, a comparatively straight knife, such as shown in Fig. 7, is better as there is less tilling resistance from the little friction area of the blade.

On most paddy fields where there is much grass and/or straw, a comparatively curved knife such as shown in Fig. 6 serves the purpose best with a minimum tilling resistance.

. The entwining of grass is effected by the edge curve and the rotational speed of the blades because the rotation will convey a centrifugal force to the grass and straw.

Fig. 8 shows a value of  $\alpha$  as when the blade holder direction is located on the X-axis and the radius is from 22 to 25 cm. With a minimum rotation of about 150 r.p.m.,  $\alpha$  should be about 60° for better performance with



Fig. 8.

the least entwining of grass and straw.

#### 3) Shapes of sections of blades

The cross-sectional shape of the blades has some influence on the torque characteristics resulting from tilling resistance. This is because most of the frictional resistance between the blades and the untilled soil is determined by the shape of the blade-cross section.



Fig. 9 shows the example section of the knife. The friction between the already tilled soil and the blade is not as great because of the freely moving action of the uncohesive soil clods and the particles. The friction which occurs between the untilled soil and the blade is therefore much greater. This friction then is variable depending on the soil conditions and the shape of the bladecross sections.

Fig. 10 shows the different shapes of bladecross sections available on the market. The friction-loss percentage is 40 to 60 per cent of the total tilling resistances in case of b in Fig. 10. The blade shapes shown by the lower figures designated as a', b', c', and d'are considered better than those illustrated above.



Fig. 10. Shapes of sections of blades

# Dynamic characteristics of rotary tilling resistances

1) Tilling resistances of a rotary knife

(1) of Fig. 11 shows the torque curves of a rotary knife mounted at the most outer position of the rotary shaft. The shape of the resistance is almost triangular shape that, the peak torque is existing on the point of one-third of the total tilling time t, that is about three-fifths of one rotation of the rotary blade.

(2) of Fig. 11 shows the torque curves of a rotary knife mounted at the inside position

of the outer knife. The peak value of the resistance is about half of the resistance of the outer knife. The total working time t is about one-third of one rotation time T of the rotary blade. The peak torque exists on the point at one-fourth of the working time t.

2) Total tilling resistances of a rotary shaft

These resistances are so variable according to soil conditions, kinds and arrangement of the rotary knives, machine structures and the tilling methods, etc., that each of the torque characteristics curve measured is different, even if total resistance of the rotary shaft is measured by using the same machine.

Fig. 12 shows a general concept of a measuring rotary-shaft torque installed in the rotary blades.

General characteristics are as follows:

- Peak torque is approximately 1.5 times mean torque.
- (2) 2 to 3 peak torques are in one rotation.
- (3) Each peak torque contains 2 to 3 sharp fluctuations.

When a tractor is tilling with maximum ability (soil hardness,  $H_{\max}$ ,  $v_{\max}$ ), "measured mean torque  $T_{u}$ " will equal the capacity of "mean torque T of engine hp".



(Test condition) Compacted clayey loam, water content=33-36% R=23cm, H=18cm,  $\beta$ =85°10',  $\beta$ 1=60° Cutting width=3cm, 1959, Sakai

Fig. 11. Torque characteristics of a knife



(Test condition) center-drive type, 16 rotary knives clayey hard soil, width of cultivation=45cm depth of cut=15cm, 7 ps Power Tiller by Sakai

Fig. 12. General concept of a rotary shaft torque installed in rotary blades

$$T_M = \eta \frac{T}{\xi}$$

 $\eta = \text{efficiency}$  $\xi = \text{reduction ratio}$ 

When the value of measured maximum ability is less than that of engine hp., this data mean that, the whole engine power of this tractor cannot be used because of some reasons.

The reasons are "excessive engine hp" or "bad balancing of the tractor's whole structure".

Machine-balancing theory presented in the previous JARQ will be available to make the reasons easily understood.

#### Summary

In this report, three important principles for designing rotary knives are analyzed and explained with the conceptional experiment data of rotary knives.

One of them is angle  $\beta_1$ , which is the rake angle between the rotor radius direction and the tangential line of tip-outside surface of the knife.  $\beta_1$  can be designed with the value of more than 45° to less than 80°, which gives throwing action to the clods, depending on soil and machine conditions.

The other is the edge curve of the knife. This has influences on the entwining of grass and straw around the blades and rotary axle, and the tilling resistance. The angle  $\phi_1$  between the radius direction and the tangential line to the edge curve is important to know its characteristics. Less entwining occurs when  $\phi_1$  is more than 57° and greater entwining occurs when  $\phi_1$  is less than 57°.

The third point is the section shape of the knife. This is related to the value of the frictional resistance between the knife and the untilled soil.

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