Theoretical Approach to the Hand Tractor of Rotary Tillage

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In 1956, the author started his agricultural engineering research on a hand tractor with a rotary tiller at Kyushu University, Japan. The research was then carried out under the advisorship of the late Prof. Shuroku Mori, (Ph.D.).

The purpose of the research was to get the design principles for a better hand tractor equipped with a rotary tiller. This machine is called "rotary power tiller" in Japan.

The research was reported by the author to the society conventions, and in 1961, all the results of the studies and researches were systematized and concised into a dissertation. This was submitted as a Ph.D. thesis to the university, and it was approved in 1962. It happened that, Japan was on a historical experience of replacing traditional man-animal-powered farming into a small scale hand-tractor-mechanized farming. This was from 1955 to 1967.

This research on rotary power tiller, as a concised dissertation was published as a book[4], in 1962. The research obtained recognitions from the Japanese Society of Agricultural Machinery, and was given the society award in 1964.

Omitting detailed analysis, the contents of the research will be divided into a few parts in series in the JARQ. Part I will be introduced in this issue of the Quarterly.

Machine stability of rotary tillage

When a tractor is performing rotary tillage, the machine shows various motion characteristics. The machine motion in case of a free-body is analyzed with the idea of arranging external forces acting on the machine from outside as the same idea as motion analysis of a hand tractor for plowing as shown in some text books[1),6)]. Discussion about the motion characteristics of rotary power tillers has to be undertaken also after knowing the effects of all external forces to the machines.

1) Arrangements of the external forces

Fig. 1 shows all external forces acting on the rotary power tiller at hand-free operation. As shown in the figure, the external forces can be divided into three groups; the center of gravity, wheels and rotary tiller.

(1) External forces on the center of gravity

Machine weight, W kg, on the gravity center of which the relative location is determined with l, h_w and θ as shown in Fig. 1. Acceleration along the forward or backward direction of the machine motion produces acceleration resistance ±A kg on the gravity center.

$$A=\frac{W(l+e)}{g}\frac{dv}{dt} \qquad (1)$$

e: correction factor of rotating mass of the machine[2)]
g: gravity acceleration, 980 cm/sec²
dv/dt: acceleration, cm/sec²

(2) External forces on the wheels

The rotary power tiller has, in general, two front drive-wheels and a rear wheel. Each wheel has a vertical load R kg and a rolling resistance f kg that are the reaction forces of soil to the wheels on the contact area. The center point of the area is indicated approximately with the clearance of ε cm as shown

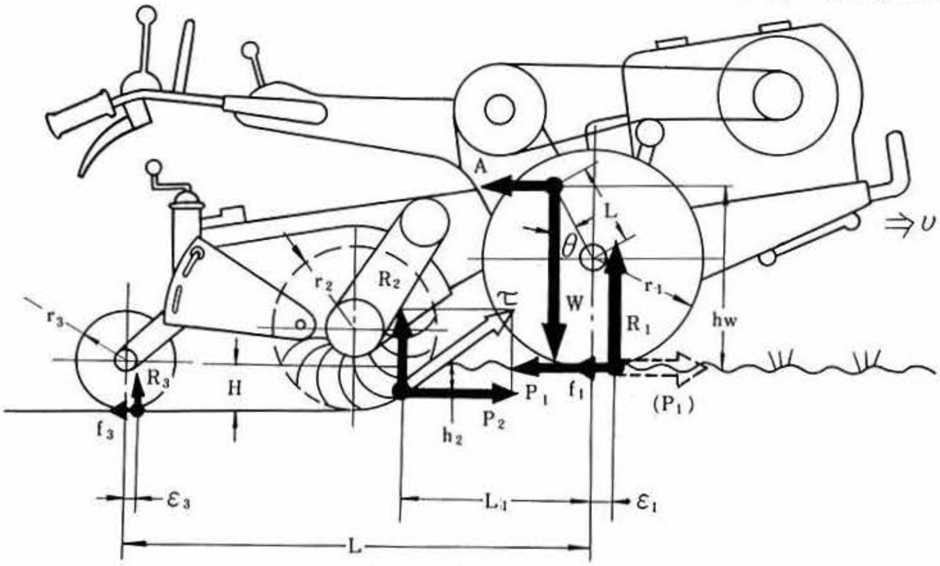

Fig. 1. External forces acting on a rotary power tiller

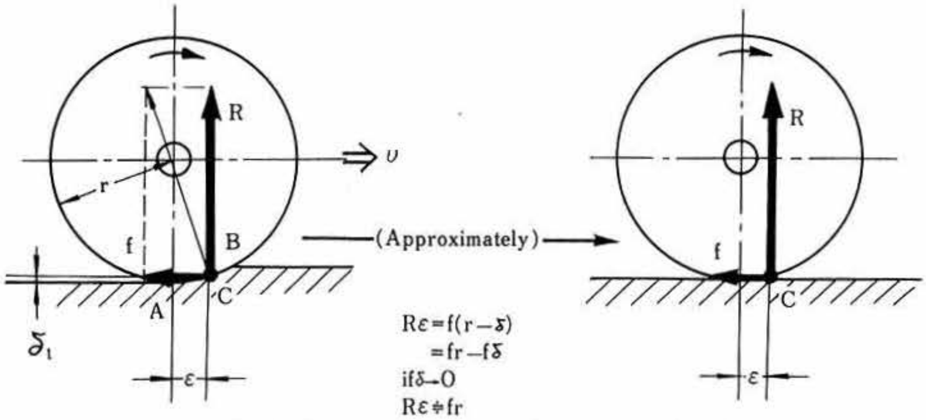

Fig. 2. Soil reaction forces of soil to the wheel

in Fig. 2 and Fig. 1.

$$f=\mu R$$

$$\varepsilon=\frac{fr}{R}=\frac{\mu Rr}{R}=\mu r \qquad (2)$$

μ: Coefficient of rolling resistance, 0.05 to 0.2

r: Radius of the wheel, cm

The drive wheel has a drive force which produces a horizontal thrust P kg as the reaction force of the soil to the wheel. P will be sometimes minus value as described below.

(3) External forces on the rotary tiller

Although there is a reaction force of total tillage resistance including suction effect of the rotary knives[5], the external forces of rotary knives in the soil can be separated into two effects of vertical and horizontal component forces R_2 kg and P_2 kg of a total tilling resistance τ kg[3].

R_2 acts so as to push up the machine. The vertical component force R_2 is named "lifting force" of the rotary tiller. When the machine is tilling with all its abilities, the value of R_2 will be as follows:

$$R_2=\frac{71620\phi_1 N_e \rho_e}{N\xi(r_2-iH)} \qquad (3)$$

N_e: Engine horsepower, ps

ρ_e: Transmission efficiency from the engine to the rotary shaft, 0.80 to 0.85

N: Engine rpm
ξ: Reduction ratio to the rotary shaft
$N\xi$: Rotary shaft rpm of the rotary tiller
r_2: Radius of rotary knives, cm
H: Depth of tillage, cm, maximum value can be adopted
71620: Constant
i: Location coefficient of the center of rotary tilling resistance

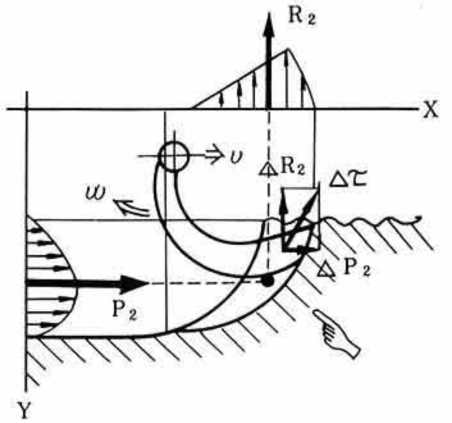

Fig. 3. The center point of rotary tilling resistances

As shown in Fig. 3, each tillage resistance $\Delta\tau$ can be separated into horizontal component ΔP_2 and vertical component ΔR_2. All ΔP_2's and ΔR_2's are arranged and summarized individually as P_2 and R_2 on X and Y axis. Projecting back of P_2 and R_2 into the soil, crossing point of both forces can be obtained. The author named this "Center Point of Rotary Tilling Resistances"[3] as the same concept as the center point of plowing resistances in the farm machinery. "i" indicates how much distance inside is the center point from trochoid locus curve of rotary knives. "i" shall be 0.1 to 0.05 of H. Dr. Shöne[7] and others also studied and proposed the location of the center point of rotary tilling resistances. θ is named "Coefficient of Lifting Force" that is from 0.7 to 1.2, depending on tillage condition.

The horizontal component force P_2 of total tilling resistance is named "Pressing Forward Force" of the rotary tiller. Having the same idea of coefficient named "Coefficient of Pressing Forward Force" θ_2, P_2 is calculated by the following equation, similar as R_2;

$$P_2=\frac{71620\phi_2 N_e \rho_e}{N\xi(r_2-iH)} \qquad (4)$$

Free-body analysis on horizontal and vertical directions

1) Machine balancing on horizontal direction

Basically, the algebraic sum of all horizontal forces acting parallel to the direction of travel must be equal to zero. When the machine is travelling at a constant speed, the average amount of A can be zero. Then, in case of shallow tillage, $f_1+f_3=P_1+P_2$ as shown in Fig. 4. However, in case of deep tillage, with all engine horsepower, Pressing Forward Force P_2 will become much bigger than the sum of f_1 and f_3. P_1 will become minus value as,

$$f_1+f_3+P_1=P_2 \qquad (5)$$

as shown in Fig. 5.

2) Machine balancing on vertical direction

The sum of all vertical forces acting perpendicular to the direction of travel must be equal to zero. The machine is supported by

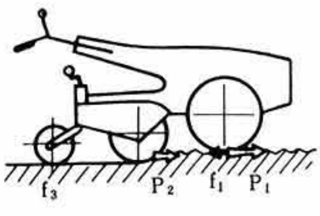

Fig. 4

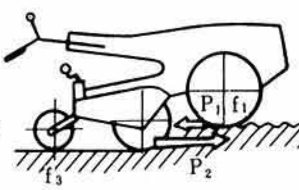

Fig. 5

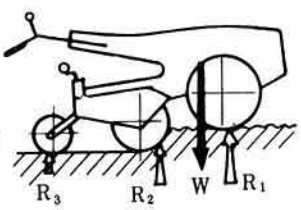

Fig. 6

three points; the front wheels, rear wheel and rotary knives as shown in Fig. 6.

$$W=R_1+R_2+R_3 \quad (6)$$

Free-body analysis of the turning moments of all external forces

The algebraic sum of the turning moments of all external forces about the contact point of the drive wheel to the ground, which is just under the front drive axle, is as follows:

$$R_2L_1+R_3(L-\varepsilon_3)+f_3H = Wl\sin\theta+Ah_W+R_1\varepsilon_1+P_2h_2 \quad (7)$$

Solving equation 7 for R_2, and substituting some equations, following the principles, the resulting equation is:

$$R_2=R'_2=\frac{Wl\sin\theta}{L_1}+\frac{Ah_W}{L_1}+\frac{M_1+M_3}{L_1}+\frac{P_2h_2}{L_1}+\frac{R_3L+f_3H}{L_1} \quad (8)$$

where:

$$M=fr=\mu Rr$$

R_2 is considered upward-resistance force of the soil being tilled to the machine, and there ought to be surely existing opposite force R'_2 of the machine to the soil in order to keep stable tilling at a constant depth.

Equation 8 shows that the rotary power tiller is receiving upward—resistance force of tillage from the soil through rotary knives, and in order to have a balanced travel, keeping a constant depth of tillage, the machine is naturally producing the downward force. This downward force is to press down the machine to the soil, which consists of five sorts of operation as shown in the right side of equation 8.

R'_2 is named "Pressing Down Force"[3] of the rotary power tiller.

1) First pressing down force

$$R'_{2a}=\frac{Wl\sin\theta}{L_1} \quad (9)$$

This is calculated with the factor as the relative situation of the center of gravity of the machine to the front wheel axle and machine weight. Equation 9 can be analyzed as follows:

$$R'_{2a}=\frac{Wl\sin\left(\theta_1+\sin^{-1}\frac{H}{X}\right)}{\sqrt{X^2-H^2}-\frac{2}{3}r_2} \quad (10)$$

X: horizontal distance between the front drive wheel axle and rotary shaft of the rotary tiller, cm. at $H=0$

Actual values for the rotary power tiller of 5 to 13 ps in the market, are 25 to 45 kg.

2) Second pressing down force

$$R'_{2b}=\frac{Ah_W}{L_1} \quad (11)$$

This force is calculated with the factors as the inertia force of tractor weight and of the rotating mass in the machine and so forth. Thus, the second pressing down force operates when travel speed of the machine changes, i.e. starting, stopping, etc. of the machine.

$$R'_{2b}\leqq\frac{\left\{r_1+l\cos\left(\theta_1+\sin^{-1}\frac{H}{X}\right)\right\}W(\rho_1-\mu_1)\times\left\{1-\frac{l\sin\left(\theta_1+\sin^{-1}\frac{H}{X}\right)}{\sqrt{X^2-H^2}-\frac{2}{3}r_2}\right\}}{\sqrt{X^2-H^2}-\frac{2}{3}r_2} \quad (12)$$

ρ_1: Coefficient of adhesion of the drive wheels, 0.6 to 0.9

This force becomes more or less equal to 100 to 200 kg in case of 5 to 13-ps machine. However, in case of constant and stable tillage, average value of this force can be considered zero.

3) Third pressing down force

$$R'_{2c}=\frac{M_1+M_3}{L_1} \quad (13)$$

This is calculated with the factors as moment effects of rolling resistances of the wheels.

$$R'_{2c}=\frac{f_1r_1+f_3r_3}{L_1}=\frac{\mu_1R_1r_1+\mu_3R_3r_3}{L_1} \quad (14)$$

Actual loads, R_1 and R_3 of the front wheels and the rear wheel should be influenced by changing the value of lifting force R_2 of the rotary tiller. Then, this force is:

$$R'_{2c}=\frac{\mu_1 r_1\left\{W-\dfrac{71620\phi_1 N_e\rho_e}{N\xi(r_2-iH)}\right\}}{\sqrt{X^2-H^2}-\dfrac{2}{3}r_2} \qquad (15)$$

Calculated values will become 20 to 30 kg in case of 5 to 13 ps machines.

4) Fourth pressing down force

$$R'_{2d}=\frac{P_2 h_2}{L_1} \qquad (16)$$

This is calculated with the factors as the "pressing forward force" of the rotary tiller. P_2 will be about 70 to 150 kg. Then:

$$R'_{2d}=\frac{143240\phi_2 N_e\rho_e H}{3N\xi\left(\sqrt{X^2-H^2}-\dfrac{2}{3}r_2\right)(r_2-iH)} \qquad (17)$$

This will be about 10 to 50 kg in case of 5 to 13 ps machines.

5) Fifth pressing down force

$$R'_{2e}=-\frac{R_3L+f_3H}{L_1} \qquad (18)$$

This is calculated with the factors as the soil reaction forces against the rear wheel.

$$R'_{2e}=-\frac{W\left\{l\sin\theta_1+\mu_1 r_1\left(1-\dfrac{l\sin\theta_1}{X}\right)\right\}}{2X} \qquad (19)$$

This will be -15 to -20 kg in case of 5 to 13 ps machines.

Whole view and its application for design principles

1) Concepts of machine balancing to cultivation

When the machine starts, the "pressing down force" R_{21}' of the machine will be as follows:

$$R'_{21}\leqq\frac{Wl\sin\theta_1}{X}+\frac{(r_1+l)W(\rho_1-\mu_1)}{X}\times\frac{r_1\mu_1W\left(1-\dfrac{l\sin\theta_1}{X}\right)}{X} \qquad (20)$$

For example: The machine of 7 ps and 60-cm width of tillage, has automatically and naturally about 216 kg or less of "pressing down force" of the rotary knives to the soil. The mark $\leqq$ in the equation 20, means effect of main clutch operation as a human factor.

When the machine is tilling at a stable constant depth, the "pressing down force" of the tiller to the soil will be as follows:

$$R'_{211}\fallingdotseq\frac{Wl\sin\left(\theta_1+\sin^{-1}\dfrac{H}{X}\right)}{\sqrt{X^2-H^2}-\dfrac{2}{3}r_2}+\frac{\mu_1 r_1\left\{W-\dfrac{71620\phi_1 N_e\rho_e}{N\xi(r_2-iH)}\right\}}{\sqrt{X^2-H^2}-\dfrac{2}{3}r_2}+\frac{143240\phi_2 N_e\rho_e H}{3N\xi\left(\sqrt{X^2-H^2}-\dfrac{2}{3}r_2\right)(r_2-iH)} \qquad (21)$$

For Example:

W : 350 kg	θ_1 : 10°	N : 1500 rpm
X : 55 cm	r_1 : 30 cm	μ_1 : 0.1
H : 18 cm	ξ : 0.15	ϕ_1 : 0.71
r_2 : 23 cm	N_e : 7 ps	ϕ_2 : 0.71
l : 10 cm	ρ_e : 0.8	i : 0.06

$$R'_{211}\fallingdotseq 42+23+17=82\text{ kg}$$

This value of R'_{211} is the mean value of "pressing down force" of the machine that shall be expected to receive the average value of the same or less "lifting force" R_2 from the soil.

2) Application for design principles

There is also a balancing equation in the horizontal direction for total machine weight W, as follows:

$$W=\frac{71620N_e\rho_e}{N\xi(r_2-iH)}\left(\phi_1+\frac{\phi_2}{\mu_1+\rho_1}\right) \qquad (22)$$

This equation means that "pressing forward force" P_2 of the rotary tiller has to be balanced with the negative effects coming from the vertical loads R_1 and R_3 of the front drive wheels and rear wheel.

W calculated by the equation 22 is used for planning design to decide the necessary weight of a rotary power tiller. However, when the soil reaction force R_2 becomes bigger than R'_{2II}, the machine cannot keep the expected depth of tillage. In such case, width of tillage can be usually adjusted. Adding these, the machine specifications and dimensions in the equations 21 and 22 can be checked so as to have a bigger "pressing down force" R'_{2II}.

In the next stage, necessary auxiliary weights, mounted to the reasonable position in the machine structure, can be calculated from the equations, when the machine is estimated to have difficulty to produce optimum "pressing down force" from changing specifications and dimensions.

Summary

Machine balancing theory of a free-body analysis was adopted to the rotary power tiller. The machine has naturally pressing down effect of the rotary power tiller. The pressing down force consists of five groups. By knowing these, necessary weight of the rotary power tiller can be calculated by adjusting main dimensions and specifications during the term of planning design. Machine performance of the rotary tiller can be promoted by theoretical calculation.

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