

A Simple Variance Estimation of Herbage Mass Based on Two Assessments Per Pasture

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Abstract

Grassland managers should consider the mean and variance of herbage mass for appropriate pasture management. Herein, a time and labor saving method to estimate variance in herbage mass was developed through extending an existing simple estimation method for mean mass. Herein, the difference between the highest and lowest masses in a given paddock and the quotient of pasture size by plot size were substituted for the range and sample size in an existing equation relating range to standard deviation (SD). The applicability of the method was evaluated using Monte Carlo simulations representing virtual pastures with a variety of conditions, and field experiments were conducted to confirm the simulation results. There was a clear relation between the ratio of the range to the SD and volume of generated random numbers in the simulation study, and the curve of the existing equation passed very close to the data points. Thus, the SD of herbage mass could be estimated using this equation. Although the accuracy of the estimate, which was expressed as a ratio of the actual variance of the random numbers, varied widely among simulated datasets, the average accuracy of the estimate was high; a similar result was produced under the field experiments that compares the new estimation method to the random sampling method.

Discipline: Grassland

Additional key words: gamma distribution, grassland management, labor saving method, Monte Carlo simulation, rising plate meter

Introduction

Grassland managers are interested in quantifying herbage mass to determine the amount of available forage and to measure the effects of certain management practices, like fertilization and grazing (Mannetje 2000). However, the methods that are generally used to measure herbage mass require time and labor; thus, there is a need to resolve these issues to enhance assessments of herbage mass.

In my previous study (Nakagami 2016a), I proposed a simple method for making approximate on-farm estimates of mean herbage mass by modifying a study completed by Iwasaki (1976). The procedure involved: (i) a visual assessment to select two sampling locations (plots) that have the highest and lowest herbage mass, respectively, in a paddock; (ii) assessment of the mean herbage mass of each plot using a Rising Plate Meter (RPM); (iii) calculation of the standardized difference between the highest and lowest mass by dividing the

difference by the sum of the two values; (iv) deriving a correction factor from the standardized difference using a single non-linear equation; and (v) estimating mean herbage mass by multiplying the average mass of the two plots by the correction factor. The method had the advantage of requiring relatively little effort (only two direct measurements per paddock), the easy selection of the plots for assessment, and acceptable accuracy of the estimation for on-farm use: more than 90% of the estimates had error rates below 20% of the true mean in the simulation study, and approximately half of the estimates had error rates below 20% of the observed mean of random samples collected in the field experiment ($R^2 = 0.71$).

Grassland managers should consider both the mean and variance of herbage mass for appropriate pasture management because pastures with large mean herbage mass might not always have a large amount of available herbage (Tsutsumi et al. 2002). For instance, even though there are many patches having tall herbage surrounding

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dung in a pasture, animals do not consume this herbage because of the smell of the dung. However, numerous assessments are normally required to measure variance in herbage mass in a given pasture, which are time and labor intensive.

By extending the previously proposed method for measuring mean herbage mass (Nakagami 2016a), it might be possible to estimate their variance. The standard deviation (SD) could be estimated from a range of random samples (Snedecor and Cochran 1967, Hozo et al. 2005, Wan et al. 2014). Therefore, the difference between the highest and lowest herbage mass, which is used to estimate mean mass (Nakagami 2016a), could potentially be used to estimate the variance of the mass in place of a range of random samples. Iwasaki (1976) showed that the SD of herbage mass might be reasonably estimated from the difference between the highest and lowest mass using Tippett's d_2 :

$$\sigma = \frac{R}{d_2} \quad (1)$$

where R is the range and σ is the SD. Tippett's d_2 is an expected value of the relative range (R/σ), and depends on the sample size only (Montgomery 2008). However, the investigation by Iwasaki (1976) was conducted in small (0.1 ha) pastures, and the formulas for d_2 are complicated and difficult to implement (Barbosa et al. 2013).

In addition to ensuring the validity and applicability of the estimation method at the on-farm level, it should be easy to implement. Wan et al. (2014) proposed a relatively simple formula to estimate the SD from the range and sample size; thus, this formula could be used to replace Tippett's d_2 in the estimation by Iwasaki's (1976) method. However, while the frequency distribution of herbage mass per unit area is skewed (Shiyomi et al. 1983, 1984), the reported relationship between SD and range assumes that data follow a normal distribution (Wan et al. 2014). Therefore, it is necessary to investigate whether the difference between the highest and the lowest mass could be used to estimate the variance in mass with sufficient accuracy.

This study aimed to establish a time and labor saving method to estimate the variance in herbage mass, even if it sacrifices accuracy to some extent. To achieve this goal, the applicability of the method to estimate variance in herbage mass by assigning the difference between the highest and the lowest mass to the equation by Wan et al. (2014) was evaluated. The Monte Carlo simulation was used to represent virtual pastures under a variety of conditions in this investigation, and field experiments were conducted to confirm the simulation results.

Materials and methods

1. Simulation study

The relationship of the variance in herbage mass to the difference between the highest and the lowest mass values was analyzed using a Monte Carlo simulation that used random numbers under the assumption that the frequency distribution of herbage mass per unit area could be described by a gamma distribution (Shiyomi et al. 1983, 1984). The dataset used here was essentially the same as the one used in my previous study (Nakagami 2016a). In brief, a set of gamma random numbers was generated for all possible combinations of the following five conditions: (1) mean herbage mass, μ ; (2) ratio of variance to mean mass at the 0.5-m² quadrat level, α ; (3) pasture size, A ; (4) plot size (surface area in which herbage mass was quantified), S ; and (5) the decrease in the variance of mass with increasing plot size, b . Two parameters, α (the shape parameter) and β (the scale parameter), with mean = $\alpha\beta$ and variance = $\alpha\beta^2$, required for generating gamma random numbers were derived from the following two equations:

$$\begin{aligned} \beta &= \frac{\sigma_s^2}{\mu} \\ \alpha &= \frac{\mu}{\beta} \end{aligned} \quad (2)$$

where σ_s^2 is the variance of herbage mass for a given plot size and can be derived as follows (Smith 1938; McIntyre 1978):

$$\begin{aligned} \sigma_s^2 &= \frac{\sigma_{05}^2}{X^b} \\ \sigma_{05}^2 &= \mu\alpha \\ X &= \frac{S}{0.5} \end{aligned} \quad (3)$$

where σ_{05}^2 is the variance of herbage mass for the 0.5-m² quadrat level, and X is plot size per unit area (0.5 m²). The volume of generated random numbers was defined as the rounded quotient of pasture size ($A \times 10,000$ m²) divided by plot size (S).

The values defined for the variables are summarized in Table 1. The mean (μ) and variance (derived from μ and α) were defined based on the values obtained from random sampling that was conducted in the field experiment described in the following section, covering most of each range of the mean and variance in herbage mass in the field experiment. The values for b were defined using a previously reported average, 0.4 (McIntyre 1978). The defined values of the current study

differed from those of the previous study (Nakagami 2016a) with respect to plot size because plot size $\geq 64 \text{ m}^2$ had been recommended to estimate mean herbage mass accurately from the comparison of plots that were 4, 16, 64, and 144 m^2 in size.

Within a set of generated random numbers, both the variance and the difference between the maximum and minimum values were calculated. A set of procedures was repeated 1,000 times for all possible combinations of variables ($5 \times 5 \times 5 \times 2 \times 3 = 750$). The mean values from each set of 1,000 iterations were used to analyze the relationship between the variance and the difference between the two values. This information was used to determine whether the equation of Wan et al. (2014), relating SD to range, could be applied to the random numbers representing herbage mass in a pasture. The equation by Wan et al. (2014) is

$$\sigma \approx \frac{R}{2\Phi^{-1}\left(\frac{n-0.375}{n+0.25}\right)} \quad (4)$$

where n is the sample size (corresponding to the volume

of random numbers); $\Phi^{-1}(z)$ is the inverse function of the cumulative distribution function of the standard normal distribution, or, equivalently, the upper z th percentile of the standard normal distribution. The simulation was performed using R version 3.1.1 (R Core Team 2014).

2. Field experiments

I used the same field experiment data as those in my previous study (Nakagami, 2016a). The experiments were conducted in pastures that were exposed to three stocking regimes: rotational, alternate, and set stocking. The dominant grass species, livestock, and management practice of each pasture is summarized in Table 2. Variance estimation in herbage mass was performed 1-5 times for each paddock by the following two methods from May 2013 to November 2013, regardless of whether the paddock was stocked or rested.

(1) Estimating the variance in herbage mass using the equation of Wan et al. (2014).

For each paddock, the maximum and minimum values of mass were the same as those in my previous study (Nakagami 2016a); that is, the mean compressed height in the areas (approximately $100\text{-}150 \text{ m}^2$; assuming

Table 1. Variables defined in the simulation study

Name	Description	Unit	Defined value
μ	Mean herbage mass	g DM m^{-2}	25, 65, 105, 145, 180
a	Ratio of variance to mean mass for a 0.5-m^2 quadrat	–	5, 30, 55, 80, 105
A	Pasture size	ha	0.5, 1, 3, 7, 12
S	Plot size (surface area in which herbage mass was quantified)	m^2	64, 144
b	Decrease in variance of mass with increasing plot size	–	0.2, 0.4, 0.6

DM: dry matter

Table 2. Study sites and their stocking management regime

Stocking regime	Location	Number of comprising paddocks	Area of paddocks (ha)	Dominant species†	Livestock
Rotational stocking	Asama Dairy Farm, Gunma, Japan (36°27'N, 138°35'E)	9	1.4-4.0	KB, TI, RCG, <i>Carex</i> spp.	One herd of 60-90 Holstein heifers was rotationally stocked using whole paddocks in the pasture for 2-4 nights every 3 weeks, approximately
Alternate stocking	Miyota Research Station, Institute of Livestock and Grassland Science, NARO, Nagano, Japan	6	0.6-2.6	KB, OG, TF	Three herds of 7-10 Japanese Black cattle were stocked using two paddocks each with a 1-week interval
Set stocking	(36°21'N, 138°29'E)	3	1.0-5.0	KB, OG, RCG, ZJ	Three herds of 5-10 Japanese Black cattle were stocked at each paddock

†KB: *Poa pratensis* L.; TI: *Phleum pratense* L.; RCG: *Phalaris arundinacea* L.; OG: *Dactylis glomerata* L.; TF: *Festuca arundinacea* Schreb; ZJ: *Zoysia japonica* Steud

125 m² in the following analysis), containing the visually selected highest and lowest mass in each paddock, was assessed using an electronic RPM (36-cm diameter, 315 g plate weight; Farmworks Ltd., Feilding, NZ). The RPM was used to obtain readings at 9-10 points at intervals of approximately 5 m (3 points × 3 rows or 5 points × 2 rows) in each area. The RPM reading was converted to herbage dry weight at 5 cm above ground level using pooled calibrations where the coefficients vary as a function of individual sampling dates (Nakagami & Itano 2014, Nakagami 2016b).

After setting the negative mass values to zero, the difference in mass between the two selected areas in each paddock was used to estimate the variance in herbage mass using the equation by Wan et al. (2014) by using the quotient of pasture size divided by assessed plot size (≈ 125 m²) in place of the sample size.

(2) Estimating variance based on the random sampling method

A compressed height within a random 0.5 m² (0.7 m × 0.7 m) area in each 12-30 m grid square constructed in each paddock was assessed using RPM and was converted to herbage mass using the method described in the previous section. The total number of samples in each paddock in the rotational, alternate, and set stocking systems was approximately 60, 60, and 100, respectively. The variance estimated by this method is hereafter referred to as the observed variance.

(3) Comparison of estimated and observed variance

The variance in herbage mass in a given paddock differs with the size and shape of the sampling quadrat or plot (Smith 1938, McIntyre 1978). Furthermore, the area that was assessed by the two methods differed (i.e., 125 m² vs. 0.5 m²). Consequently, a direct comparison of the two values would provide incorrect results. Therefore, the usefulness of the estimate using the equation by Wan et al. (2014) was evaluated by making a relative comparison of the two estimates. To achieve this objective, each value obtained from the estimated and observed variance was standardized by divided by the respective mean variances; hereinafter they are referred to as the standardized estimated variance and standardized observed variance, respectively.

Results

1. Simulation study

(1) Relationship between the variance and the range

The volume of generated random numbers ranged from 35 (*A* = 0.5 ha, *S* = 144 m²) to 1875 (*A* = 12 ha, *S* = 64 m²) in this simulation. The Monte Carlo analysis showed a clear relationship between the ratio of the range

to the SD (*R*/*σ*) and the volume of generated random numbers, when using the average of 1,000 iterations. Even though there was a certain level of longitudinal variance in the scatter points caused by variation in the combinations of the simulated conditions, the relationship was considered acceptable, regardless of the defined conditions, except when the minimum value was ≤ 1 g dry matter (DM) m⁻² (Fig. 1). The curve of the equation by Wan et al. (2014) passed very close to the data points. These results indicate that the SD of herbage mass can be estimated by substituting the difference between the highest and lowest mass and the quotient of pasture size divided by plot size for the *R* and *n* in the equation of Wan et al. (2014), respectively.

(2) Accuracy and precision of the estimates from the equation of Wan et al. (2014)

For all simulation data, the variance of each dataset was estimated from the equation of Wan et al. (2014). Although the mean ratio of the estimate to the actual variance of the random numbers was close to 1, the ratio varied widely among cases; i.e., only 70% of cases had a ratio within 0.8-1.2, and approximately 20% of the cases had a ratio of more than 1.2 (Table 3). Although this result occurred regardless of the simulation conditions, the accuracy and precision at a minimum value of ≤ 1 g DM m⁻² was lower than those in the other cases.

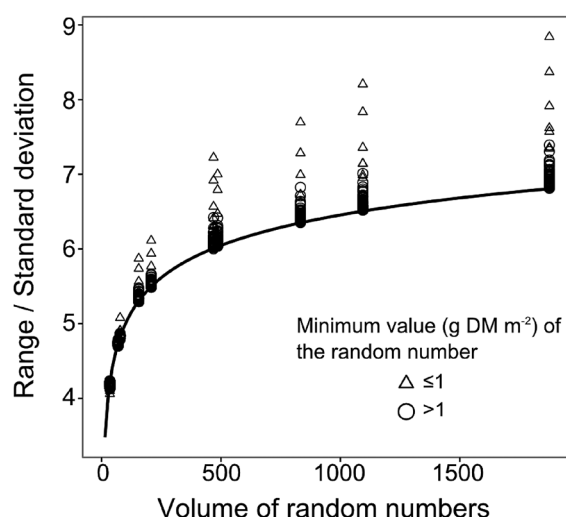


Fig. 1. The volume of generated random numbers affects the relationship between the range and standard deviation (SD) of random numbers in the simulation.
The solid line indicates the equation of Wan et al. (2014) (see text).

Table 3. Accuracy of the simulation estimates

Defined herbage mass (g DM m ⁻²)	Volume of simulation data (× 1000)	Accuracy of estimation (Estimated variance / actual variance)	Probability within each level of ratio (%)		
			< 0.8	0.8–1.2	> 1.2
Minimum value > 1 g DM m ⁻²					
25	108.3	1.03 ± 0.23†	11.2	69.4	19.4
65	145.4	1.04 ± 0.22	10.1	70.9	19.0
105	149.9	1.03 ± 0.21	9.8	72.3	18.0
145	150	1.03 ± 0.21	9.3	73.6	17.1
185	150	1.03 ± 0.21	9.1	74.2	16.6
All	704	1.03 ± 0.22	10.2	71.2	18.6
Minimum value ≤ 1 g DM m ⁻²					
25	41.7	1.23 ± 0.43	9.5	47.7	42.8
65	4.6	1.19 ± 0.34	6.0	56.0	38.0
105	0.05	1.11 ± 0.23	3.6	72.7	23.6
145	0	-	-	-	-
185	0	-	-	-	-
All	46.4	1.23 ± 0.42	9.1	48.6	42.3
Whole data					
25	150	1.09 ± 0.31	10.7	63.4	26.0
65	150	1.04 ± 0.23	10.0	70.5	19.6
105	150	1.03 ± 0.21	9.8	72.3	18.0
145	150	1.03 ± 0.20	9.3	73.6	17.1
185	150	1.03 ± 0.20	9.1	74.2	16.6
All	750	1.04 ± 0.24	9.8	70.8	19.4

†Mean ± SD. Data include all possible combinations of the simulation conditions, except for defined herbage mass.

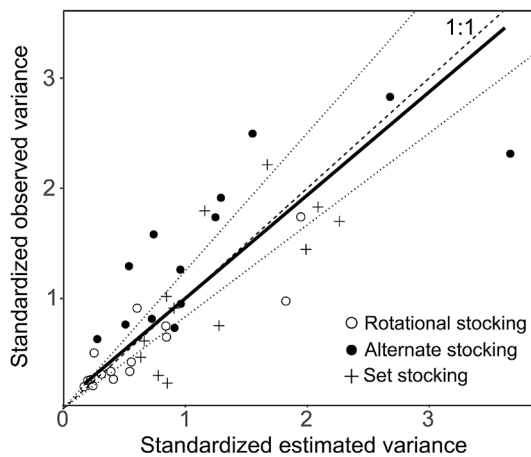


Fig. 2. Relationship between the observed variance in herbage mass based on a random sampling method and the estimated variance based on the mass in two plots containing the highest and the lowest mass, respectively, under different stocking systems in the field experiment

The variances were standardized by dividing by the respective mean variance. Dashed line represents $y = x$; dotted lines represent $y = x/0.8$ and $y = x/1.2$; bold line represents the fitted major axis regression line ($y = 0.06 + 0.94x$, $r = 0.81$).

2. Field experiments

The quotient of pasture size divided by plot size ranged from 49 ($A = 0.6$ ha, $S \approx 125$ m²) to 335 ($A = 4.2$ ha, $S \approx 125$ m²). The results of the relative comparison between the standardized estimated variance using the equation of Wan et al. (2014) and the standardized observed variance from random sampling is shown in Figure 2. The scatter points were spread randomly on either side of the 1:1 line. The major axis regression equation between the estimated and the observed values was $Obs = 0.06 + 0.94 Est$, with a correlation coefficient of 0.81. Thus, the accuracy of the estimation was high on average. However, accuracy varied widely among measurements, with only 34% of measurements having a ratio of the estimate to the observed variance within 0.8–1.2. The remaining 32% and 34% of cases had ratios of less than 0.8 and more than 1.2, respectively.

Discussion

In this study, a time and labor saving method to estimate variance in herbage mass was developed. The estimation procedure involved: (i) selecting two sampling

locations (plots) of approximately 100-150 m² that had the highest and lowest herbage mass, respectively, in a paddock by visual assessment; (ii) measuring the mean herbage mass of each plot using an RPM or other suitable methods; (iii) calculating the difference in mass between the two plots; and (iv) estimating the variance of herbage mass by substituting the difference in mass and the quotient of pasture size by plot size for the R and n in the equation of Wan et al. (2014), respectively.

The procedures (i) and (ii) are used in common with the methods for estimating the mean mass in my previous study (Nakagami 2016a). Although this method requires information about pasture size and plot size, the former is generally known and the latter could be assumed to be 100-150 m². Consequently, the only assessment required in each paddock is that of the highest and lowest mass. Thus, this method is labor saving. Although the equation of Wan et al. (2014) seems complicated, because it includes the upper z th percentile $\Phi^{-1}(z)$, this value is easily computed, for example, by the command “qnorm (z)” in statistical software R or the function of “NORM.INV” in Microsoft Excel®.

Although the estimation method was not always precise (i.e., the accuracy varied widely among cases), the estimates were considered to be acceptable as an approximate on-farm indicator of the effectiveness of pasture management, particularly based on the ease of implementation of the method.

1. Relationship between the variance and the range

The strong relationship between the statistical range and SD is well documented (Snedecor and Cochran 1967) and the relationship is affected by sample size; i.e., proportionality coefficients depend on sample size (Snedecor and Cochran 1967, Hozo et al. 2005, Wan et al. 2014), as well as Tippet's d_2 (Montgomery 2008).

The relationship between the ratio of the range to SD and the volume of random numbers was very clear, regardless of the combination of the defined simulate conditions, except when the minimum value was ≤ 1 g DM m⁻² (Fig. 1). The results were versatile, because the simulation covered a wide range of conditions (i.e., mean, variance, and plot size) that were defined based on the values obtained from actual pastures. The exceptions (the minimum value was ≤ 1 g DM m⁻²) arose when the defined mean herbage mass, μ , was 25 g DM m⁻², which rarely occurs in an actual grazing situation, and when the decrease in the variance of the mass with increasing plot size, b , was 0.2, which was only half of the empirical mean. The equation of Wan et al. (2014) fitted the data well, indicating that the equation could be applied to quantify the variance in herbage mass of pastures.

2. Accuracy and precision of the estimation

In the simulation, the mean accuracy of the estimated variance was high, with the mean ratio of the estimate to actual variance being 1.04, as in the case with the accuracy of the estimated mean mass in my previous study (Nakagami 2016a). However, the precision of the estimation for variance was low, with only 70% of cases having error rates below 20% (Table 3). This result differed from the precision of the estimated mean mass, in which more than 90% of cases had error rates below 20% (Nakagami 2016a). This difference was attributable to the fact that, while the average of iterations formed a clear curve, individual datasets spread widely around the curve. Furthermore, the variation among datasets was larger for the variance in the generated random numbers than it was for their mean. Specifically, the coefficient of variation (CV) of the variance for 1,000 iterations was 2.7-48.6 times (mean: 9.1 times) larger than that found for the mean. Furthermore, the ratio of the respective CVs of variance and mean had a strong positive correlation with the square root of the shape parameter in the gamma distribution ($r = 0.99$, $P < 0.001$, $n = 750$). This result indicates that the lower precision obtained in the present study was caused by the property of the gamma random number. Therefore, it would be difficult to increase the precision of the estimate, even with an improved equation that includes other measurable variables. Thus, the proposed method currently represents the best approach.

In the field study, because of the difference in the size of the sampling plots (i.e., variance in herbage mass depends on plot size), along with the fact that larger plots having smaller variance (Smith 1938, McIntyre 1978), the estimated and observed estimate were standardized by dividing by the respective mean variance, allowing relative comparison. The comparison showed that the mean accuracy of the field data was sufficiently high, as well as the mean accuracy in simulated data, but the accuracy of individual measurements varied more widely than in the simulation (Fig. 2).

One of the factors affecting the low estimate precision in the field study was the error in plot selection. If an assessed plot does not have the appropriate highest and lowest mass, the derived difference in mass between the two plots is smaller than the appropriate difference, leading to variance in herbage mass being underestimated. This issue was reduced by setting the size of assessment plot as a space of 100-150 m², instead of assessing a point of 0.5-1 m². The selection for the appropriate plots that had the highest and the lowest mass was easier for plots that covered a large area as compared to plots that covered a small area, because there would be many candidate assessment plots of 1m² level in a pasture, but only a few

candidate plots at the 100m² level. The selection in this case would be more straightforward and appropriate by using an aerial overview image of a pasture taken by a plane or unmanned aerial vehicle.

In addition, the correctness of the measurement of the mass in each assessed plot also affects the estimation precision. Mass was relatively homogeneous in the plots, with a reading of only 9-10 of RPM being required to measure the mass in each plot. However, the presence of different dominating plant species among plots led to incorrect estimates of mean mass, because of differences in the relationship between the RPM reading and the mass of different plant species.

On the other hand, the observed variance, which was based on 60-100 random samples, was also not always precise. The observed variance had wide confidence interval (CI); i.e., the mean width of the 95% CI was approximately 3,000, and was equivalent to 70% of the mean observed variance. Thus, numerous random assessments might not always provide sufficient data for precise pasture management. Therefore, the estimate precision of the method proposed here, which estimates variance in herbage mass using only two assessments for a given pasture, was considered viable.

3. Applicability of the method

The greatest advantage of this method is that little effort is required, as only two assessments must be made for the estimation. The standard approach for estimating variance requires numerous assessments for a given pasture. This approach is time-consuming because, even if non-destructive tools like RPM are used, it is necessary to record each reading manually when automated logging techniques for such readings do not exist. Besides simple random sampling, some effective procedures have been developed for estimating the variance in herbage mass. For example, the ranked-set sampling method requires fewer samples (MacEachern et al. 2002), but still far more than the two samples used in the method that is reported here. Estimation methods based on the gamma model (Shiyomi 1991, Tsutsumi & Itano 2005, Itano et al. 2006) require only two direct harvest measurements, but are based on dozens of assessments by visual or other non-destructive methods.

Conversely, the disadvantage of this method was its low precision, which was mainly caused by errors in the selection and measurement of assessment plots. Furthermore, the accuracy of the results cannot be stated objectively, because the method involves intentional, non-random sampling (Neyman 1934). Therefore, this method should be used after due consideration of these limitations.

The estimate of the variance value of herbage mass by the method reported here can be used as a guide for pasture management by combining with the mean value of the mass estimated by the method of previous research (Nakagami 2016a). Although individual measurements might have substantial error, estimates of overall seasonal trends and annual variation from periodic measurements could be used to indicate the effectiveness of existing pasture/grazing management strategies, such as fertilization, stocking density, stocking cycle, and the need for pasture renovation.

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