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Digital Filters to Eliminate or Separate Tidal Components in Groundwater Observation Time-Series Data

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Abstract

This paper discusses digital low-pass filters for application to tidally fluctuated groundwater observation data. Three types of filters that are commonly used, mainly for oceanography, and newly produced filters are comparatively evaluated with a focus on their ability to eliminate major diurnal and semidiurnal tidal components. All the digital filters presented are the nonrecursive type that can easily be used with spreadsheet software. Newly produced low-pass filters are excellent tide-killer filters with a length of 241 hours applicable to hourly sampled time-series data. The new filters suppress eight major diurnal and semidiurnal tides to practically negligible magnitudes (10^{-8} order input), with longer-period components (longer than two days) being nearly completely preserved. High-pass filters transformed from these new tide-killer low-pass filters can separate the components of semidiurnal to diurnal tidal periods from other longer-period components, keeping approximately the same magnitude as in the input data for eight major tides. Therefore, the use of the new high-pass filters prior to quantitative analysis of major tidal components in groundwater observation data should effectively improve the accuracy of analysis.

Discipline: Agricultural engineering
Additional key words: comparative study, nonrecursive filter, tidal filter, low-pass filter, high-pass filter

Introduction

There are many islands and coastal areas where agriculture represents a principal economic sector, and the inhabitants rely on groundwater for domestic and agricultural use, because surface water is often limited in their geological settings. In such areas, strategies for the sustainability and development of agriculture, along with the incorporation of appropriate groundwater resource management are needed (Van der Velde et al. 2007, Nawa & Miyazaki 2009, Ishida et al. 2011, Yoshimoto et al. 2011, Duncan 2012, Kobayashi & Koda 2012, Baharuddin et al. 2013, Koda et al. 2014). Shirahata et al. (2014) estimated hydraulic properties of an unconfined aquifer on an island where groundwater development was desired for agricultural use. The observation data used for the estimation exemplified groundwater fluctuations affected by periodic tides and other non-periodic agents. An example of non-periodic signals in the observation data was the effect of a few day-long drops in atmospheric pressure when a typhoon passes, an effect caused through the intermediary of changes in height of hydraulically connected surface water for small permeable islands (Vacher 1978). After continuous groundwater observations are made in insular or coastal areas, elimination or separation of tidal components from other components in the collected data is generally an important preliminary step to investigate and evaluate groundwater resources.

Observation time-series data including significant tidal signals are often encountered as a matter of course in geophysics and hydrology, as well as hydrogeology. In the fields of geodesy and oceanography, digital filters have been commonly used to smooth time-series data. These filters suppress tidal components in the observation data and

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disclose lower-frequency (longer-period) signals. Nevertheless, application of digital filters to groundwater tidal fluctuations is limited (e.g., Lam 1974, Serfes 1991, Sánchez Úbeda et al. 2015).

This paper discusses digital low-pass filters for suppressing major tidal components in observation time series, focusing on types that can be immediately used with a prevalent spreadsheet application. It describes the results of a comparative study on commonly used digital filters in the literature, mainly but not limited to oceanography. In addition, this paper reports and compares newly produced excellent filters. The types of low-pass digital filters dealt with in this paper can be transformed into high-pass filters through an arithmetic procedure. High-pass filters derived from the new low-pass filters are excellent for extracting the components of semidiurnal to diurnal tidal-period ranges separately from longer-period components, without alterations in the major tidal constituents. The digital filters presented may provide convenient tools for investigating groundwater resources in many insular areas and countries.

Nonrecursive digital filtering and filter response

An introduction to digital filtering is given by Duchon & Hale (2011, 143-182), and more advanced explanations are provided by Emery & Thomson (2001, 514-554). This section summarizes an introduction to the key points on the filter types of interest.

There are two general types of digital filters. One type is the recursive filter in which current output is related to input and the past values of output. The other type is the nonrecursive filter in which the output can be obtained only using first input data. All filters presented in this paper are the nonrecursive type and can explicitly operate with a spreadsheet application through built-in functions.

Suppose we have observation time-series data of the following sequence:

$$..., x_{-1}, x_0, x_1, x_2, ..., x_n, ... \quad (x_n = x(t_n),\ n \text{ is integer}), \tag{1}$$

with observations at discrete time $t_n = t_0 + n \cdot \Delta t$ where t_0 is the origin of time and Δt the sampling interval. This paper only deals with symmetric filters that use an odd number of consecutive input data points. Symmetry is thus required to preserve the phase information of the data. A nonrecursive digital filter has a sequence with $(2m + 1)$ weight factors:

$$W_{-m}, W_{-(m-1)}, ..., W_{-1}, W_0, W_1, ..., W_m \tag{2}$$

that satisfy the symmetry requirement $W_{-k} = W_k$. Filtering is represented by:

$$y_n = \sum_{k=-m}^{m} W_k \cdot x_{n+k} \tag{3}$$

where y_n is the output. This calculation, sum of products of corresponding components of two number sequences, can be made directly with the help of a built-in function of spreadsheet software.

Digital filter performance is specified by the filter response factor or function:

$$R(\omega) = W_0 + 2 \cdot \sum_{k=1}^{m} W_k \cos(\omega k \cdot \Delta t) \quad \text{or} \tag{4}$$

$$R(T) = W_0 + 2 \cdot \sum_{k=1}^{m} W_k \cos((2\pi/T) k \cdot \Delta t). \tag{5}$$

The filter response factor indicates the output amplitude of the signal of a particular angular frequency (ω) or period (T) when an input signal of the same frequency (or period) has unit amplitude. A low-pass filter exhibits responses near unity at low frequencies (long periods), but nearly zero at high frequencies (short periods). In other words, the filter response function has a passband at a low-frequency, long-period part and a stopband at a high-frequency, short-period part (see graphical examples given later in this paper). The transition between the bands can be represented by a "cutoff frequency." For practical filters, the transition covers a range of frequencies, and the cutoff frequency is defined as the frequency at which the amplitude preserved in the passband is decreased by a factor of 2 or √2 (i.e., response factor of 0.5 or 1/√2). The corresponding "cutoff period" is also used. A symmetric low-pass filter naturally satisfies the normalization requirement:

$$\sum_{k=-m}^{m} W_k = W_0 + 2 \cdot \sum_{k=1}^{m} W_k = 1 \tag{6}$$

to achieve unit response for zero frequency (infinite period).

From a normalized nonrecursive low-pass filter $\{W_k\}$, a high-pass filter $\{W^h_k\}$ can be derived as:

$$W^h_0 = 1 - W_0 \quad \text{and} \quad W^h_k = - W_k \ (k \neq 0). \tag{7}$$

If the response function of the original low-pass filter is given as $R(\omega)$ or $R(T)$, the response function of the corresponding high-pass filter produced by (7) is simply:

$$R^h(\omega) = 1 - R(\omega) \quad \text{or} \quad R^h(T) = 1 - R(T). \tag{8}$$

Because a nonrecursive low-pass filter can be easily transformed into a high-pass filter, there is no need to prepare a separate high-pass filter.

When a low-pass filter perfectly exhibits zero responses at the frequencies of the major tidal constituents, it is a good “tide-killer” filter. And when such a good tide-killer low-pass filter is transformed into a high-pass filter, the high-pass filter is good for preserving tidal constituents. In other words, the output time-series data has major tidal components of exactly the same magnitude as the input data.

Comparison of digital low-pass filters

Four types of low-pass filters are presented below with their composition of weights and response factors. Three types that have been commonly used are presented together with their extended or generalized families. The other type includes excellent tide-killer filters produced as per a design procedure proposed in an oceanographic study (Thompson 1983). Input and output time series are assumed to have hourly sampling intervals, unless otherwise specified.

The desired low-pass filter in this study is one with a response factor near zero at a period range shorter than about 29 h (i.e., the range of diurnal tides and shorter), whereas the responses consistently show near unity at periods longer than two days. A length of two days is generally the central period of the oceanic “spectral gap,” and a period range longer than this (and shorter than ten days) is called a “weather band” (Emery & Thomson 2001, 405, 540). Especially for an ideal “tide-killer” filter, responses for the periods of eight major tidal constituents (K_2, S_2, M_2, N_2, K_1, P_1, O_1, Q_1) should be zero. High-pass filters are not directly discussed here, but a good tide-killer low-pass filter makes a good “tide-preserving” high-pass filter, as explained in the last part of the previous section.

1. Running-mean filters

Running mean values are obtained by the application of a digital filter consisting of an odd number ($2m + 1$) of equal weight factors:

$$1/(2m+1),\ 1/(2m+1),\ ...,\ 1/(2m+1). \tag{9}$$

This filtering is easily realized by a series of average calculations over ($2m + 1$) input-data points. Various running-mean filters can be made by choosing the positive integer m.

Figure 1 shows the response factors (plotted against the period of signals) of running-mean filters for $m = 6$, 12, and 24. The filter lengths are 13, 25, and 49 data points or h, respectively (with input-data lengths used for one output data point being the same). As seen in the upper panel of the figure, the response factors are relatively small for shorter periods (higher frequencies). However, they are generally large for longer periods (lower frequencies) and approach unity, thereby characterizing these filters as low-pass filters.

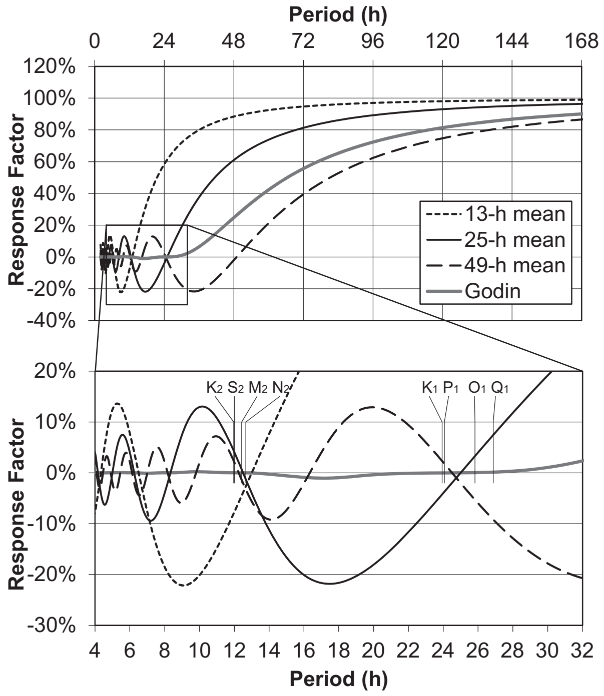

Fig. 1. Response functions of 13-, 25- and 49-h running-mean filters and a Godin filter (cascaded 24-, 24- and 25-h running means). Responses at periods shorter than 2 h are not plotted (same hereinafter)

The lower panel of Fig. 1 shows a close-up of a short-period range including the major semidiurnal (K_2, S_2, M_2, N_2) and diurnal (K_1, P_1, O_1, Q_1) tidal constituents. Periods of the eight major tides are marked in the plot area. When a running-mean filter is used to smooth time-series data by suppressing the major tides, near-zero response factors for these tidal periods are advantageous. Besides, a shorter digital filter is generally preferred because it can yield a longer output time series from a practical finite input time series with less loss of length at both ends of the input series (see Appendix). A 25-h running-mean filter is naturally chosen from various running-mean filters and used, as responses at the semidiurnal and diurnal tidal bands are both relatively small, and particularly small for the M_2 tide, the largest constituent in most cases.

Nevertheless, the 25-h running-mean filter leaves some amounts of major tidal components. Table 1 summarizes the performance of running-mean filters and other filters (explained later). Filter types are listed in the leftmost column and the 25-h running-mean filter is shown in the second row. Response factors for the eight major tidal constituents range from –4.3% to +7.4%. In addition, as already seen in Fig. 1, within the semidiurnal to diurnal tidal bands taken here as 10- to 29-h periods (including major and minor tides), the response factor fluctuates widely and

Table 1. Performances of low-pass filters for hourly sampled time series

Filter	Filter length (data points or hours)	Filter Response										Major tides							
		High-frequency band						Low-frequency band											
		T = 2–10 h		T = 10–29 h (semidiurnal to diurnal)		T = 29–48 h		T = 48–240 h (weather band)		T = 240– (–26500 h)		K_2 (ca.11.97h)	S_2 (12.00h)	M_2 (ca.12.42h)	N_2 (ca.12.66h)	K_1 (ca.23.93h)	P_1 (ca.24.07h)	O_1 (ca.25.82h)	Q_1 (ca.26.87h)
		Min	Max	Min	Max	Min	Max	Min	Max	Min	Max								
13-h running mean	13	-22.16%	13.63%	-20.1%	70.2%	70.2%	88.4%	88.4%	99.5%	99.5%	100.0%	-7.9E-02	-7.7E-02	-4.5E-02	-2.7E-02	5.8E-01	5.9E-01	6.3E-01	6.6E-01
25-h running mean	25	-9.43%	12.94%	-21.8%	15.5%	15.5%	61.0%	61.0%	98.2%	98.2%	100.0%	4.3E-02	4.0E-02	6.4E-03	-1.3E-02	-4.3E-02	-3.7E-02	3.3E-02	7.4E-02
49-h running mean	49	-5.92%	5.05%	-15.6%	12.9%	-21.7%	-2.0%	-2.0%	93.3%	93.3%	100.0%	2.3E-02	2.0E-02	-1.4E-02	-3.3E-02	2.3E-02	1.8E-02	-5.3E-02	-9.2E-02
Godin (24-24-25 running means)	71	-0.08%	0.22%	-1.0%	0.6%	0.6%	24.8%	24.8%	95.0%	95.0%	100.0%	3.2E-07	*zero*	7.9E-06	-3.8E-05	-3.2E-07	-2.8E-07	1.9E-04	1.0E-03
Doodson	39	-57.08%	10.12%	-0.1%	8.7%	3.6%	41.0%	41.0%	96.8%	96.8%	100.0%	3.3E-04	*zero*	-5.8E-04	1.7E-03	1.5E-04	-1.3E-04	3.0E-03	1.0E-02
Pertzev	37	-29.78%	74.42%	-0.1%	10.8%	3.7%	41.4%	41.4%	96.9%	96.9%	100.0%	3.8E-04	*zero*	-6.6E-04	1.9E-03	1.5E-04	-1.3E-04	3.1E-03	1.1E-02
Nakagawa (bbcc)	37	0.00%	100.00%	0.0%	7.4%	4.5%	42.7%	42.7%	97.0%	97.0%	100.0%	1.8E-05	*zero*	2.8E-03	6.5E-03	9.2E-06	9.3E-06	6.8E-03	1.6E-02
Nakagawa (ee)	41	0.00%	100.00%	0.0%	6.2%	2.7%	38.5%	38.5%	96.6%	96.6%	100.0%	3.3E-03	2.9E-03	7.1E-05	2.8E-04	2.1E-03	1.6E-03	1.2E-03	6.2E-03
Nakagawa (bde)	43	-20.00%	41.36%	-3.3%	3.7%	2.9%	38.2%	38.2%	96.6%	96.6%	100.0%	-1.6E-06	*zero*	-3.5E-05	1.6E-04	1.1E-04	-9.4E-05	2.3E-03	8.2E-03
Nakagawa (bcde)	55	-20.00%	18.88%	-2.2%	2.7%	0.8%	27.0%	27.0%	95.4%	95.4%	100.0%	1.6E-06	*zero*	3.5E-05	-1.6E-04	-4.6E-07	-4.0E-07	2.5E-04	1.4E-03
119Hanncos36h	119	-0.01%	0.01%	-0.6%	14.9%	14.9%	86.3%	86.3%	100.8%	100.0%	100.1%	6.6E-05	5.2E-05	-1.5E-04	-2.3E-04	-2.5E-04	1.0E-03	3.2E-02	6.3E-02
119Hanncos40h	119	0.00%	0.01%	-0.6%	6.2%	6.2%	74.0%	74.0%	101.1%	100.0%	100.5%	1.2E-04	1.2E-04	5.1E-05	-3.5E-05	-6.3E-03	-6.3E-03	1.3E-03	1.4E-02
119Hanncos48h	119	-0.01%	0.01%	-0.6%	0.2%	0.2%	49.8%	49.8%	100.2%	100.0%	100.2%	4.5E-06	1.7E-05	1.5E-04	1.8E-04	-1.1E-03	-1.4E-03	-5.7E-03	-6.4E-03
121Hammcos40h	121	-0.03%	0.03%	-0.3%	5.1%	5.1%	75.2%	75.2%	99.9%	99.9%	100.0%	-4.3E-04	-4.3E-04	-2.0E-04	5.0E-05	-2.9E-03	-2.9E-03	2.1E-03	1.2E-02
121Ham56cos40h	121	-0.05%	0.04%	-0.1%	4.6%	4.6%	75.8%	75.8%	99.6%	99.6%	100.0%	-7.0E-04	-7.0E-04	-3.2E-04	9.2E-05	-1.2E-03	-1.2E-03	2.5E-03	1.0E-02
239Hanncos36h	239	0.00%	0.00%	-0.6%	0.3%	0.3%	100.0%	99.8%	100.6%	100.0%	100.0%	7.0E-06	1.1E-05	1.3E-05	-1.7E-05	8.9E-04	1.1E-03	1.7E-04	-4.3E-03
239Hanncos40h	239	0.00%	0.00%	-0.4%	0.2%	-0.6%	90.7%	90.7%	100.6%	99.9%	100.0%	-1.5E-05	-1.6E-05	1.1E-05	2.0E-05	-7.7E-04	-7.7E-04	1.2E-03	1.8E-03
239Hanncos48h	239	0.00%	0.00%	-0.1%	0.1%	-0.6%	50.0%	50.0%	100.7%	100.0%	100.2%	1.2E-05	1.2E-05	-8.1E-06	-1.5E-05	3.9E-04	3.9E-04	-4.8E-04	-7.5E-04
241Hammcos36h	241	-0.06%	0.06%	-0.2%	0.4%	0.4%	99.6%	99.6%	100.1%	99.8%	100.0%	1.5E-04	2.3E-05	-6.7E-04	9.6E-05	6.9E-05	-1.6E-04	-1.3E-03	-1.6E-03
Thompson ‘120i913’	241	-0.19%	0.17%	-0.8%	4.7%	4.7%	100.5%	99.3%	100.3%	100.0%	100.1%	1.4E-04	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	2.2E-07
Hanawa&Mitsudera ‘24tk’	241	-0.19%	0.20%	-1.0%	3.0%	3.0%	99.8%	99.4%	100.6%	99.4%	100.0%	-2.9E-06	-3.5E-06	-6.0E-07	3.3E-06	7.2E-07	3.5E-06	3.6E-06	2.8E-07
LP241H079122k38i	241	-0.10%	0.10%	-0.8%	0.5%	0.5%	100.4%	99.7%	100.4%	100.0%	100.1%	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*
LP241H079122k25i	241	-0.10%	0.11%	-0.8%	0.5%	0.3%	100.3%	99.8%	100.3%	100.0%	100.1%	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*
LP241H079122kM3	241	-0.10%	0.10%	-0.6%	0.4%	0.4%	100.4%	99.8%	100.4%	100.0%	100.1%	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*	*abs<E-08*

zero : perfect zero; *abs<E-08* : the absolute value is less than 1×10^{-8}.

roughly ±20%. And the response-function line in the long-period band never becomes horizontal or straight (Fig. 1), indicating that filtering modifies the frequency composition of the long-period range. These disadvantages of the running-mean filter may not be crucial when subjecting output data to qualitative analysis. Otherwise, another digital filter that alleviates the disadvantages above as described below should be adopted.

Godin (1966) introduced cascaded running-mean filters. One filter is commonly used as a low-pass filter to remove diurnal, semidiurnal, and shorter-period components (Emery & Thomson 2001, 532). The filter consists of successive uses of three running means of 24-, again 24- and 25-h lengths. This is identical to the application of the following 71-h-long weighted filter (Thompson 1983):

$$W_k = \begin{cases} [0.5/(24^2 \cdot 25)] \cdot [1200 - (12-k)(13-k) \\ \qquad - (12+k)(13+k)] \quad (0 \le |k| \le 11) \\ [0.5/(24^2 \cdot 25)] \cdot (36-k)(37-k) \\ \qquad (12 \le |k| \le 35). \end{cases} \tag{10}$$

Both Fig.1 and Table 1 include the responses of this filter. This Godin filter exhibits improved performance as a tide-killer filter, with response factors of between ±0.004% for six major constituents (K_1, S_2, M_2, N_2, K_1, P_1), +0.02% for O_1, and +0.1% for Q_1. One clear failing of the Godin filter as a low-pass filter is its slow transition from the short-period stopband to the long-period passband, which leads to a significant reduction in non-tidal variations in the output data for periods ranging from a few days to a week.

2. Selected-mean filters

According to Parker (2007, 128), one of the earliest and most widely used tidal filters in oceanography is the Doodson filter (Doodson's 1928 formula for "X_0"). This filter has a length of 39 h and 39 weights:

$$W_k = \begin{cases} 0 \quad (k = 0, \pm 5, \pm 8, \pm 10, \pm 13, \pm 15, \pm 16, \pm 18) \\ 1/30 \quad (k = \pm 2, \pm 3, \pm 6, \pm 7, \pm 11, \\ \qquad \pm 12, \pm 14, \pm 17, \pm 19) \\ 2/30 \quad (k = \pm 1, \pm 4, \pm 9). \end{cases} \tag{11}$$

One feature of this filter is the use of zero-weighted terms, resulting in a substantial composition of 24 selected terms ("working weights") that help reduce computational labor relative to the length of the input data points.

Pertzev (1957) succeeded in substantially simplifying the Doodson filter into a 15-term selected-mean filter (Nakagawa 1961). One output value is calculated from an input series of length 37 h, and the weight factors are:

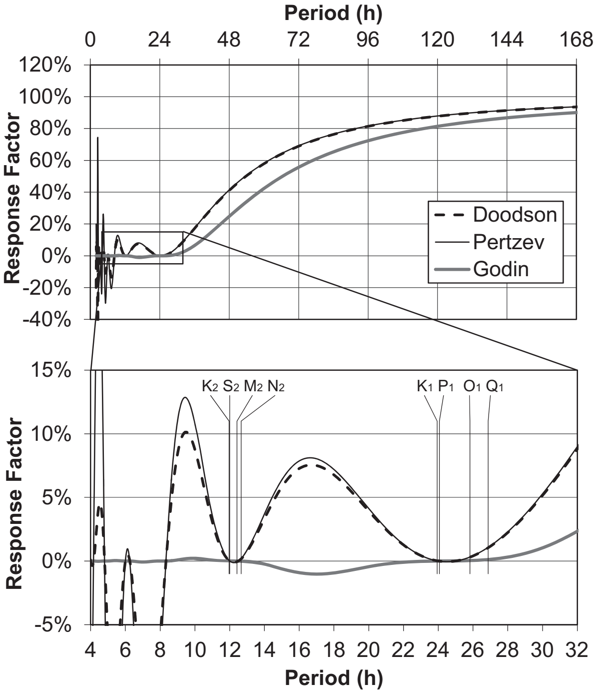

Fig. 2. Responses of Doodson and Pertzev filters compared to a Godin filter (cascaded 24-, 24- and 25-h running means). In the upper panel, Doodson and Pertzev filters nearly overlap each other

$$W_k = \begin{cases} 0 \quad (k = \pm 1, \pm 4, \pm 6, \pm 7, \pm 9, \pm 11, \\ \qquad \pm 12, \pm 14, \pm 15, \pm 16, \pm 17) \\ 1/15 \quad (k = 0, \pm 2, \pm 3, \pm 5, \pm 8, \pm 10, \pm 13, \pm 18). \end{cases} \tag{12}$$

Figure 2 shows responses of the two filters above. The two responses are almost the same for major four semidiurnal tides and for periods ranging from diurnal to longer. Melchior (1959) pointed out that this coincidence is no accident. Both filters have the same drawbacks as the Godin filter, although to a lesser extent, with slow transitions between the stopband and passband. For the eight major tides, the responses of Doodson and Pertzev filters are considerably small (Table 1). In a period range shorter than semidiurnal, the response factors widely vary beyond a range of ±50%.

Nakagawa (1961) briefly summarized the previous development of tidal filters for use in geodesy, and presented a generalized procedure for making selected-mean filters. He demonstrated a score of selected-mean filters, including the two filters above (as his (ade) and (de) filters). Four other of Nakagawa's (1961) selected-mean filters (without zero-weighted terms) are as follows:

(bbcc), using 7 selected data points from a 37-h-long input and

$$W_k = \begin{cases} 4/16 & (k = 0) \\ 3/16 & (k = \pm 6) \\ 2/16 & (k = \pm 12) \\ 1/16 & (k = \pm 18) \end{cases} \quad (13)$$

(ee), using 9 selected data points from a 41-h-long input and

$$W_k = \begin{cases} 5/25 & (k = 0) \\ 4/25 & (k = \pm 5) \\ 3/25 & (k = \pm 10) \\ 2/25 & (k = \pm 15) \\ 1/25 & (k = \pm 20) \end{cases} \quad (14)$$

(bde), using 27 selected data points from a 43-h-long input and

$$W_k = \begin{cases} 2/30 & (k = 0, \pm 5) \\ 1/30 & (k = \pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 8, \pm 10, \\ & \quad \pm 11, \pm 13, \pm 15, \pm 16, \pm 21) \end{cases} \quad (15)$$

(bcde), using 43 selected data points from a 55-h-long input and

$$W_k = \begin{cases} 3/60 & (k = \pm 1) \\ 2/60 & (k = 0, \pm 4, \pm 5, \pm 6, \pm 7, \pm 9, \pm 11) \\ 1/60 & (k = \pm 2, \pm 3, \pm 8, \pm 10, \pm 12, \pm 13, \pm 14, \pm 15, \\ & \quad \pm 16, \pm 17, \pm 19, \pm 21, \pm 22, \pm 27) \end{cases} \quad (16)$$

Figure 3 and Table 1 show the responses. Nakagawa (1961) demonstrated that various selected-mean tide-killer filters could be created to balance the performance and computational labor of filtering.

The (bbcc) and (ee) filters above may be applied automatically to over-hourly sampled data, such as (bbcc) filter to 3- or 6-hourly data, because the requisite input data are at regular intervals. However, such use of selected-mean filters should be followed by a survey with an understanding of the aliasing effect and Nyquist frequency or period (e.g., Hanawa & Mitsudera 1985; Emery & Thomson 2001, 434-438).

The shapes of the four response-function lines (Fig. 3) are similar to the Doodson or Pertzev filters. The selected-mean filters are generally good for use as tide-killer filters, but not good for use as low-pass, high-stop filters, because the selected-mean filters were primarily developed for limited computational resources.

3. Cosine filters using windows

According to Emery & Thomson (2001), the terms Lanczos-cosine filter and cosine-Lanczos filter are general names encompassing a variety of cosine-type filters using windows (smoothing or tapering functions) presented in oceanographic literature. The "cosine filter" seems to be so called in oceanography because the weights are formulated with the intention of making the response function a truncated Fourier cosine series (Emery & Thomson 2001, 535-536). The cosine filter without window has $(2m + 1)$ weight factors:

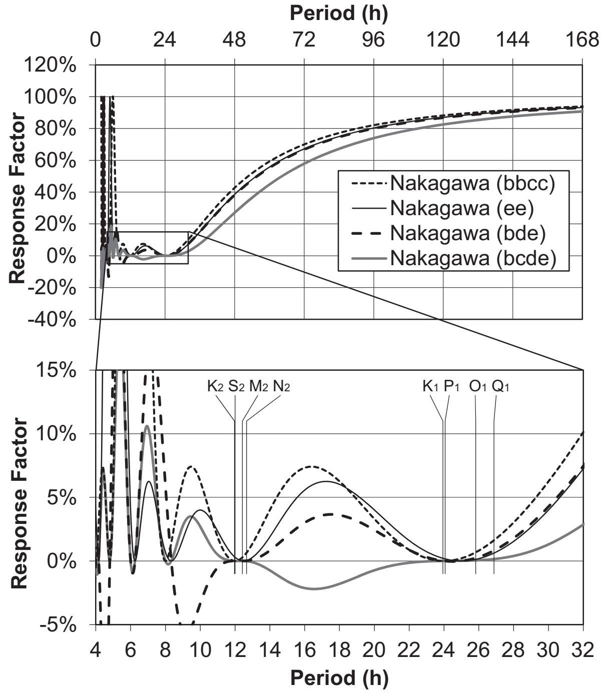

Fig. 3. Responses of Nakagawa's (1961) selected-mean filters. In the upper panel, (ee) and (bde) filters nearly overlap each other

$$W_k = \begin{cases} T_N/T_C & (k = 0) \\ (T_N/T_C)\cdot\sin(k\pi\cdot T_N/T_C)/(k\pi\cdot T_N/T_C) & \\ \quad (1 \le |k| \le m) \end{cases} \quad (17)$$

where T_N is the Nyquist period (twice the sampling interval) and T_C ($> T_N$) the cutoff period defined by a filter response factor of 0.5. This primitive cosine filter is of little use by itself, because, without a window, the response function has essentially inevitable large oscillations that are called Gibbs' phenomenon. The "Lanczos" smoothing denotes an idea of averaging out the oscillation by multiplying the weights of the cosine filter by a tapering function with a period of $2m$ with respect to k (Emery & Thomson 2001, 536-538).

A widely used Lanczos-cosine filter with a raised cosine window (also known as "von Hann," "Hann," or "Hanning" window) is the $m = 60$ version of:

$$W_k = \begin{cases} 1 & (k = 0) \\ 0.5\cdot[1 + \cos(k\pi/m)]\cdot\sin(k\pi\cdot T_N/T_C)/(k\pi\cdot T_N/T_C) & \\ \quad (1 \le |k| \le m) \end{cases} \quad (18)$$

normalized to satisfy (6) (Emery & Thomson 2001, 533-

540). The normalization is achieved by dividing each tentative weight (produced by (18)) by the sum of the $(2m + 1)$ tentative weights. Various filters can thus be made by choosing m and T_C. This filter substantially has a length of $(2m - 1)$, because two W_m at the ends are invariably zero. In a strict sense, the resulting cutoff period is not always equal to but approximated by the used T_C value, and considerably deviates from T_C for a T_C/T_N large enough to compete with m. The integer m is supposed to be preset so large as to overwhelm T_C/T_N.

For hourly sampled data with a Nyquist period (T_N) of 2 h, setting m as 60 makes a filter of 119 weight factors:

$$W_k = \begin{cases} 1 \quad (k = 0) \\ 0.5\cdot[1 + \cos(k\pi/60)]\cdot\sin(k\pi\cdot 2/T_C)/(k\pi\cdot 2/T_C) \\ \qquad (1 \le |k| < 60) \end{cases} \tag{19}$$

normalized as before. In oceanography, this Lanczos-cosine filter using the von Hann window with a cutoff period (T_C) of approximately 34.3 h (precisely, 240/7 h), proposed by Mooers & Smith (1968), has widespread application as a low-pass filter (Thompson 1983; Emery & Thomson 2001, 539). On the other hand, Thompson (1983) suggested the use of a filter with a cutoff period of 40 h (the "Lancz6" filter, in the present paper referred to as "119Hanncos40h") to emphasize the small response of diurnal tides.

Figure 4 compares the response functions of three filters generated by (19) (and normalization) with cutoff periods (T_C) of 36, 40, and 48 h, and another cosine-type filter explained a little later. First of all, relative to previous simple running-mean and selected-mean filters, the response factors for the short-period band (shorter than 29 h) are markedly improved (approach zero). For semidiurnal tides, their responses are adequately small. However, they are not perfect for suppressing diurnal tides and the cutoff period must be reluctantly chosen depending on the priority objective of the filtering intended. For example, the filter above with T_C of 36 is good for suppressing K_1 and P_1 constituents, but passes relatively large proportions of O_1 and Q_1, whereas the filter with T_C of 48 is superior for stopping diurnal tides, but will undesirably reduce long-period (over 48 h) variations in the observation data. The cutoff period of 40 h suggested by Thompson (1983) may be an appropriate compromise.

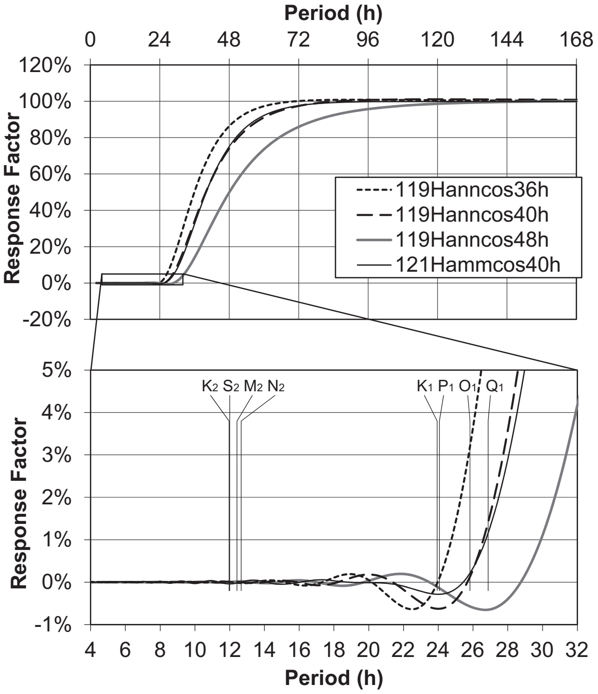

Fig. 4. Responses of 119- and 121-h-long cosine-type filters using windows. Filter names with "Hann" and "Hamm" indicate uses of the von Hann window and the Hamming window, respectively. The last part of the names denotes the cutoff period. In the upper panel, two filters with a cutoff period of 40 h nearly overlap each other. Note that the vertical scale is different from the preceding figures

Employment of the Hamming window (e.g., Emery & Thomson 2001, 446-448) in place of the von Hann window in (18), resulting in:

$$W_k = \begin{cases} 1 \quad (k = 0) \\ [0.54 + 0.46\cos(k\pi/m)]\cdot\sin(k\pi\cdot T_N/T_C)/(k\pi\cdot T_N/T_C) \\ \qquad (1 \le |k| \le m) \end{cases} \tag{20}$$

with normalization, makes other cosine-type filters. One example is "121Hammcos40h" with $m = 60$, $T_N = 2$, and $T_C = 40$, included in Fig. 4. This filter shows a little more balanced response between semidiurnal and diurnal tides (Table 1), at the expense that perfect unit response in the passband is not achieved (though 99.7% response is attained at a period of 96 h). Furthermore, other slightly different filters can be created modifying the Hamming window by replacing the coefficients {0.54, 0.46} in (20) with other pairs of numbers with sum of one. The windows created in this manner, including the von Hann window (with coefficients {0.5, 0.5}) and the original Hamming window, comprise a family of windows called "generalized Hamming window." For example, if we emphasize the balance of response between four major tides (S_2, M_2, K_1, O_1) (keeping $m = 60$, $T_N = 2$, $T_C = 40$), coefficients {0.56, 0.44} may be the best possible choice ("121Ham56cos40h" in Table 1). The filter response in the passband attains and remains over 99.2% for periods longer than 96 h. If a filter that predominantly suppresses the largest diurnal constituent (K_1) is preferred, coefficients {0.5738, 0.4262} could be adopted. The produced filter would achieve a negligible response for K_1 (absolute value $< 10^{-7}$), but give slightly large response magnitude for semidiurnal tides (e.g., a response factor of -4.1×10^{-4} for M_2). Not only the filter using the von Hann window (119Hanncos40h), but filters using generalized Hamming windows with the incorporated coefficients between {0.54, 0.46} and {0.58, 0.42} inclusive will be a possible choice, when an imperfect passband with a short-period end of about four days is acceptable (instead of two days desired in the present study).

Setting *m* as a larger value can make other cosine-type filters that exhibit a more desirable response. Figure 5 shows the responses of four cosine-type filters produced by *m* as 120. The response transition between the stopband and passband covers a narrower period range than the *m* = 60 filter of the same cutoff period. Both cosine filters with a cutoff period of 36 h using the von Hann window and the Hamming window ("239Hanncos36h" and "241Hammcos-36h" in Fig. 5, respectively) are reasonable choices, provided that the doubled filter length and slightly poor suppression near the period of 28 h are approved.

Cosine filters using windows have a stopband with a response factor smoothly approaching zero as the signal period becomes shorter. Using a sufficient filter length with an appropriate combination of a cutoff period and window can produce a good tide-killer low-pass filter.

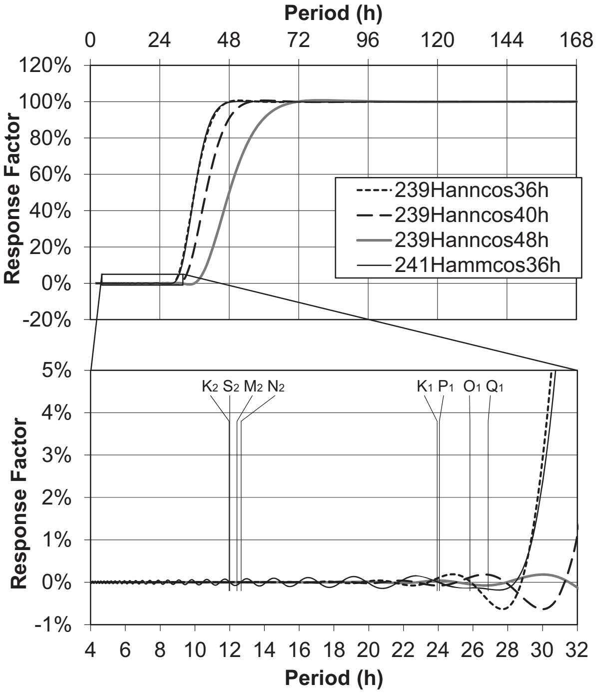

Fig. 5. Responses of 239- and 241-h-long cosine-type filters using windows. In the upper panel, two filters with a cutoff period of 36 h nearly overlap each other

4. Optimized tide-killer filters

Tide-killer low-pass digital filters were newly produced in this study as per a design method using matrix calculations proposed by Thompson (1983). The method optimizes filters according to the user's purpose. A low-pass filter is created not only specifying a desired transition range with two boundary frequencies (low-end Ω_1 and high-end Ω_2), but also imposing zero responses at arbitrary frequencies within the stopband. Thompson (1983) made tide-killer low-pass filters of various lengths from 49 to 241 h, including the 241-h-long "120i913" filter with imposed zero responses for seven major tides (S_2, M_2, N_2, K_1, P_1, O_1, Q_1) and a local inertial frequency (of Sydney, period T = ca. 21.6 h). Hanawa & Mitsudera (1985) followed Thompson's (1983) design concept and generated other tide-killer filters imposing zero responses at frequencies of eight major tides (K_2, S_2, M_2, N_2, K_1, P_1, O_1, Q_1) and also three inertial frequencies for latitudes 32.5°N, 35°N, and 40°N, in anticipating uses near Japan. Their "24tk" filter has a stopband with a long-period end near 28 h. Details of the design method and resultant weight factors are available in the original articles. The weight factors of this type of filters cannot be expressed in simple formulas.

In the present study, three new 241-h-long low-pass filters were produced with imposed zero responses for the same eight major tides as covered by Hanawa & Mitsudera (1985) and a slightly enlarged stopband width. The desired long- and short-period ends of the transition band were set at Ω_1 = 7.9 degree/h (T = ca. 45.6 h) and Ω_2 = 12.2 degree/h (T = ca. 29.5 h), respectively. Three filters—"LP241H-079122k38i," "LP241H079122k25i" and "LP241H-079122kM3"—were produced. The first was produced with an additionally imposed zero response at the inertial frequency for 38°N (T = ca. 19.5 h). The second imposed a zero response at the inertial frequency for 25°N (T = ca. 28.4 h). These inertial frequencies were not considered by Hanawa & Mitsudera (1985). The third filter was produced without any inertial frequencies, but imposed a zero response for M_3 tide (T = ca. 8.28 h). Table 2 gives the weight factors of the filters produced.

Figure 6 demonstrates the response factors for the three new filters, together with the preceding two filters of the same length and similar cutoff periods. Differences between the five are subtle, but the newly produced filters exhibit a stopband of near-zero responses wider than that of the preceding two. All new filters show negligibly small responses for the major eight tides (Table 1) and for each additional zero-imposed frequency (between $\pm 1 \times 10^{-8}$). In the pass band, response factors of all three attain and remain greater than 99.3% for periods longer than 45 h.

Thompson's (1983) method allows for settings of arbitrary filter response factors at arbitrary signal periods. The method is effective in making a low-pass filter to eliminate signals of major tidal constituents.

Table 2. Weights of newly produced tide-killer low-pass digital filters for hourly time-series data. All three filters are symmetric and have a length of 241 data points or h. Table 1 and Figure 6 show the response factors

LP241H079122k38i				LP241H079122k25i				LP241H079122kM3			
k	Wk	k	Wk	k	Wk	k	Wk	k	Wk	k	Wk
	$(\times10^{-9})$				$(\times10^{-9})$				$(\times10^{-9})$		
0	55790638			0	55802188			0	55836513		
1	55502028	61	-2893531	1	55512607	61	-2901366	1	55545013	61	-2871467
2	54641146	62	-2916094	2	54648890	62	-2932852	2	54676083	62	-2901478
3	53222852	63	-2847495	3	53226134	63	-2873922	3	53246194	63	-2840840
4	51271900	64	-2695136	4	51269517	64	-2731079	4	51282099	64	-2696472
5	48822847	65	-2468547	5	48814223	65	-2512723	5	48820073	65	-2477870
6	45919771	66	-2179105	6	45905150	66	-2229007	6	45905242	66	-2196402
7	42615707	67	-1839718	7	42596291	67	-1891725	7	42591062	67	-1864518
8	38971724	68	-1464451	8	38949692	68	-1514148	8	38938794	68	-1495190
9	35055611	69	-1068073	9	35033991	69	-1110759	9	35016644	69	-1101717
10	30940197	70	-665521	10	30922563	70	-696822	10	30898309	70	-697782
11	26701395	71	-271285	11	26691417	71	-287763	11	26660809	71	-297482
12	22416085	72	101247	12	22416998	72	101610	12	22381837	72	84988
13	18160001	73	440443	13	18174074	73	457968	13	18137036	73	436145
14	14005743	74	736987	14	14033860	74	770279	14	13997718	74	744403
15	10021044	75	984288	15	10062474	75	1030455	15	10029311	75	1001253
16	6267353	76	1178644	16	6319760	76	1233701	16	6290591	76	1202007
17	2798744	77	1319143	17	2858447	77	1378494	17	2833408	77	1345794
18	-338875	78	1407285	18	-276450	78	1466196	18	-297495	78	1434812
19	-3108314	79	1446395	19	-3048066	79	1500378	19	-3064891	79	1473113
20	-5481482	80	1440923	20	-5428061	80	1485997	20	-5439817	80	1465396
21	-7439654	81	1395756	21	-7396974	81	1428608	21	-7402480	81	1416211
22	-8973643	82	1315689	22	-8944547	82	1333763	22	-8942988	82	1329764
23	-10083856	83	1205145	23	-10069973	83	1206728	23	-10061594	83	1210230
24	-10780204	84	1068195	24	-10781959	84	1052539	24	-10768345	84	1062273
25	-11081774	85	908848	25	-11098520	85	876328	25	-11082235	85	891432
26	-11016217	86	731505	26	-11046409	86	683774	26	-11030129	86	704196
27	-10618802	87	541426	27	-10660172	87	481494	27	-10645716	87	507767
28	-9931134	88	345031	28	-9980817	88	277170	28	-9968581	88	309672
29	-8999587	89	149885	29	-9054219	89	79311	29	-9043285	89	117410
30	-7873544	90	-35696	30	-7929349	90	-103395	30	-7918232	90	-61796
31	-6603573	91	-203408	31	-6656499	91	-263033	31	-6644137	91	-221236
32	-5239687	92	-345959	32	-5285621	92	-393519	32	-5272106	92	-355101
33	-3829794	93	-458104	33	-3864886	93	-491522	33	-3851560	93	-459125
34	-2418456	94	-537456	34	-2439532	94	-556988	34	-2428414	94	-531386
35	-1045985	95	-584878	35	-1050981	95	-593106	35	-1043844	95	-572995
36	252126	96	-604285	36	263790	96	-605646	36	266208	96	-588258
37	1445488	97	-601878	37	1472688	97	-601784	37	1470930	97	-584035
38	2508891	98	-584909	38	2548820	98	-588638	38	2544305	98	-568297
39	3422279	99	-560246	39	3470725	99	-571838	39	3464935	99	-548218
40	4170675	100	-533051	40	4222555	100	-554457	40	4216261	100	-528402
41	4744124	101	-505869	41	4794176	101	-536567	41	4787111	101	-509864
42	5137659	102	-478377	42	5181181	102	-515531	42	5172320	102	-490138
43	5351268	103	-447839	43	5384765	103	-486972	43	5373012	103	-464465
44	5389820	104	-410195	44	5411436	104	-446205	44	5396330	104	-427669
45	5262888	105	-361508	45	5272549	105	-389755	45	5254578	105	-376041
46	4984424	106	-299410	46	4983677	106	-316616	46	4964044	106	-308669
47	4572253	107	-224167	47	4563848	107	-228890	47	4543850	107	-227838
48	4047395	108	-139068	48	4034710	108	-131669	48	4015103	108	-138531
49	3433223	109	-50009	49	3419656	109	-32137	49	3400418	109	-47308
50	2754541	110	35669	50	2742985	110	61855	50	2723620	110	39029
51	2036614	111	110793	51	2029105	111	143416	51	2009349	111	114637
52	1304244	112	170031	52	1301814	112	207991	52	1282335	112	175516
53	580939	113	211159	53	583658	113	254149	53	566318	113	220005
54	-111782	114	235568	54	-104630	114	283551	54	-117152	114	248903
55	-754927	115	247822	55	-744611	115	300082	55	-749706	115	265186
56	-1332407	116	254321	56	-1320532	116	308390	56	-1316632	116	273278
57	-1831262	117	261378	57	-1819611	117	312195	57	-1806766	117	277864
58	-2241760	118	273197	58	-2232199	118	312857	58	-2211967	118	282408
59	-2557388	119	290282	59	-2551813	119	308605	59	-2526530	119	287725
60	-2774772	120	308773	60	-2775047	120	294715	60	-2746844	120	291115

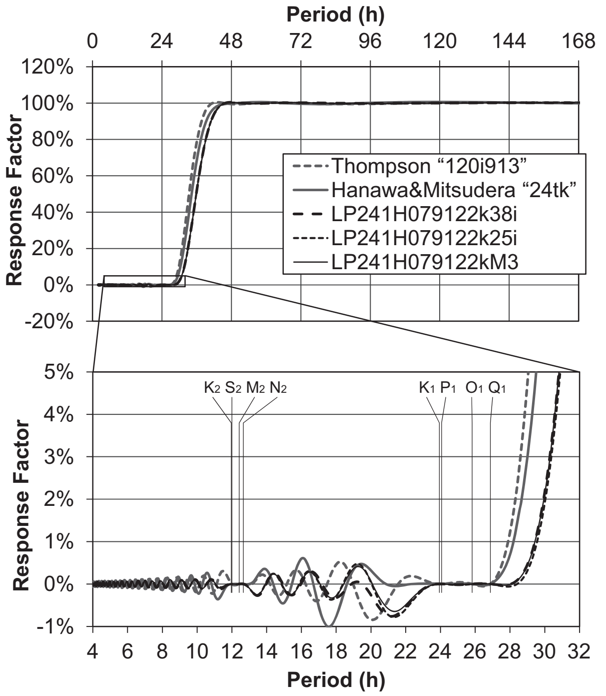

Fig. 6. Responses of tide-killer low-pass filters produced by Thompson's method (1983). Thompon's (1983) "120i913" and Hanawa & Mitsudera's (1985) "24tk" and three filters newly produced in the present study. A presumable typographic error in the weight factor table for "24tk" is corrected beforehand (W_{34} should have a negative sign). In the upper panel, the three new filters nearly overlap each other

Discussion

Various digital filters have been developed and used to remove tidal effects from observation data in oceanography and geodesy. At present, filters can be chosen depending on particular situations and personal preferences or familiarities (Emery & Thomson 2001, 515; Parker 2007, 128-129).

Running-mean filters are the simplest low-pass filters to apply to digital observation data on a spreadsheet. When hourly sampled observation data with tidal signals are to be investigated, the 25-h running-mean filter can be adopted to visually smooth the time-series graph and reveal an outline of longer-period variations. Selected-mean filters are also available and generally have higher ability to reduce tidal signals than the simple running-mean filters. Both types of filters are not good for the accurate preservation of long-period fluctuations for a few days to a week, although deformation effects in that period range may not be detected by visual inspection of a time-series graph (see Fig. A1).

The windowed cosine filters demonstrated in this paper are good for suppressing semidiurnal tides, but relatively poor for diurnal tides. With a sufficient length and an appropriate cutoff period, however, these filters could prove to be satisfactory tide-killer low-pass filters.

As a tide-killer low-pass filter, Thompson's filters are superior to the cosine-type filters of a similar length. The filters newly produced in the present study are superior in terms of eliminating the major eight tidal constituents. Although the purpose of this study was to improve groundwater data analysis, the filters produced will be applicable to various tidally fluctuated observation data, such as data collected from the surface water of oceans or estuaries, as well as coastal groundwater. The new filters also appear excellent, when transformed as high-pass filters to separate diurnal and semidiurnal tidal bands without loss or unwanted gain of major tidal constituents. These high-pass filters are perfect for use prior to the simple harmonic analysis proposed by Shirahata et al. (2014) that extracts specific major tidal constituents from groundwater data, because the filters remove monotonic trends and long-period components that may increase harmonic-analysis errors (Shirahata et al. 2016).

Concluding remarks

A variety of tidal filters, including some not presented in this paper, easy to apply to time-discrete digital data are available. At least for low-pass and high-pass filters applied to field observation data subject to limited accuracy, the filters demonstrated (ranging from simple to sophisticated) should include a sufficient variety. We leave to future work any case studies of the application of these filters to real groundwater observation data, but the digital filters will certainly contribute to the appropriate development or management of precious groundwater resources in many developing insular countries.

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Appendix: Demonstration of filtering effects

Figure A1 demonstrates the effects of some low-pass filters presented in the main text and the corresponding high-pass filters produced by Equation (7). The original data is a month-long time-series plot of hourly sea-level observation data published on the web by the Japan Meteorological Agency (ISHIGAKI site, July 2014). The output time-series length is shorter than the input series by approximately the filter length.

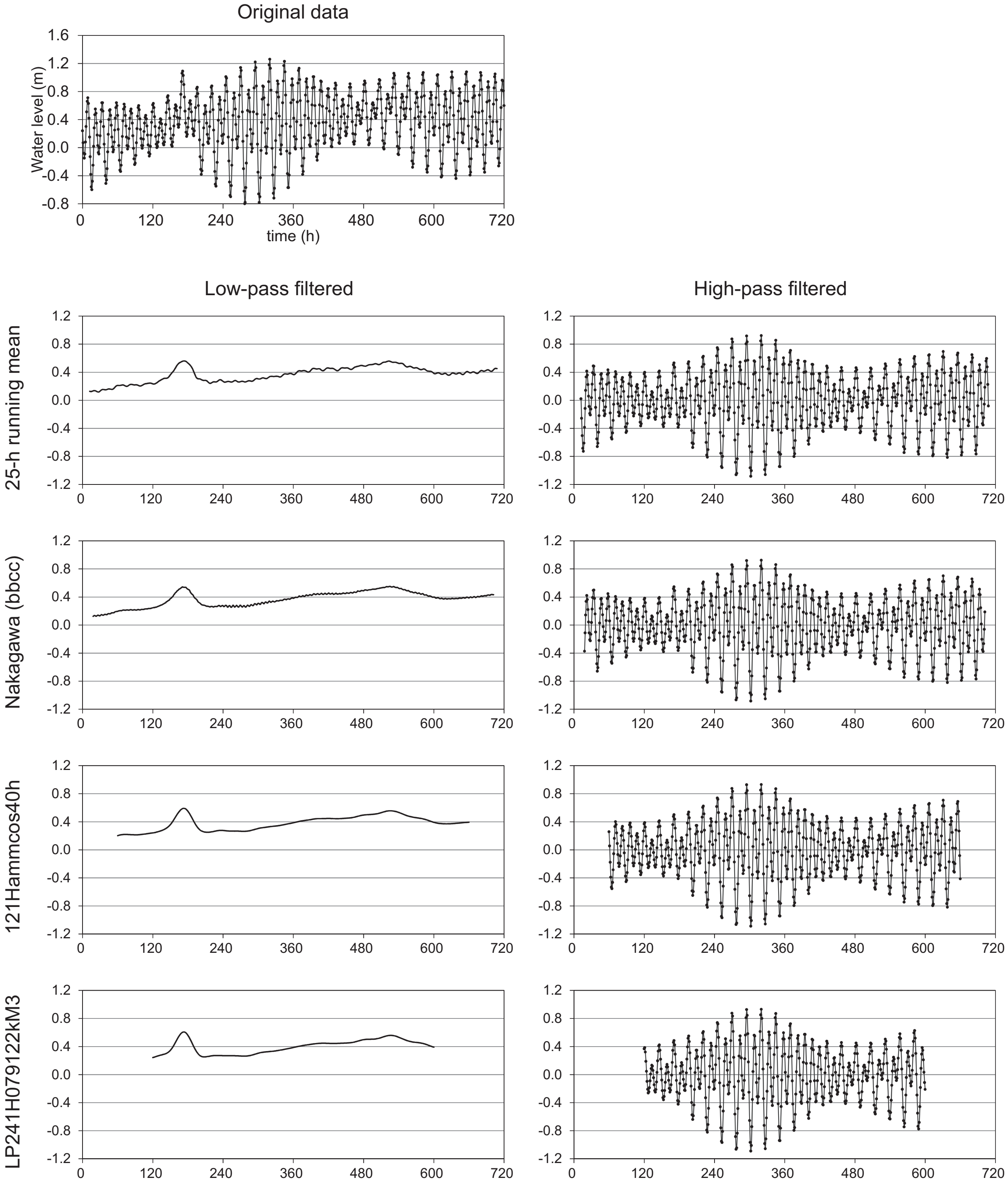

Fig. A1. Effects of low-pass filters and high-pass filters. These filters are the 25-h running-mean filter, Nakagawa's (bbcc) selected-mean filter, 121-h-long cosine filter using the Hamming window with a cutoff period of 40 h, newly produced LP241H079122kM3, and corresponding high-pass filters

References

Baharuddin, M. F. T. et al. (2013) Assessment of seawater intrusion to the agricultural sustainability at the coastal area of Carey Island, Selangor, Malaysia. *Arabian Journal of Geosciences*, **6**, 3909-3928.

Doodson, A. T. (1928) The analysis of tidal observations. *Philosophical Transactions of the Royal Society of London. Series A*, **227**, 223-279.

Duchon, C. & Hale, R. (2011) *Time Series Analysis in Meteorology and Climatology: An Introduction*. John Wiley & Sons, New Jersey, USA, pp.262.

Duncan, D. (2012) *Freshwater Under Threat / Pacific Islands /*

Vulnerability Assessment of Freshwater Resources to Environmental Change. United Nations Environment Programme (UNEP), pp.58.

Emery, W. J. & Thomson, R. E. (2001) *Data Analysis Methods in Physical Oceanography (Second and Revised Edition)*. Elsevier, Amsterdam, Netherland, pp.638.

Godin, G. (1966) Daily mean sea level and short-period seiches. *International Hydrographic Review*, **43**, 75-89.

Hanawa, K. & Mitsudera, H. (1985) On the data processings of daily mean values of oceanographical data: Note on the daily mean sea-level data. *Engan kaiyo kenkyu noto* (*Bulletin on Coastal Oceanography*), **23**, 79-87 [In Japanese].

Ishida, S. et al. (2011) Sustainable use of groundwater with underground dams. *Japan Agricultural Research Quarterly*, **45**, 51-61.

Japan Meteorological Agency: *Choseki kansoku shiryo Ishigaki* (Tide observation data Ishigaki). http://www.data.jma.go.jp/kaiyou/db/tide/genbo/genbo.php [In Japanese].

Kobayashi, T. & Koda, K. (eds.) (2012) Development of survey method for freshwater lens in Marshall Islands. *JIRCAS Working Report*, **77**, pp.69.

Koda, K. et al. (2013) Estimation of hydraulic parameters in the freshwater lens aquifer in Laura Island. *Suido no chi* (*Water, Land and Environmental Engineering*), **81**, 541-545 [In Japanese].

Lam, R. K. (1974) Atoll permeability calculated from tidal diffusion. *Journal of Geophysical Research*, **79**, 3073-3081.

Melchior, P. J. (1959) Sur l'interprétation des courbes de dérive des gravimètres. *Bulletin d'Informations Marées Terrestres*, **17**, 279-287 [In French].

Mooers, C. N. K. & Smith, R. L. (1968) Continental shelf waves off Oregon. *Journal of Geophysical Research*, **73**, 549-557.

Nakagawa, I. (1961) General considerations concerning the elimination of the drift curve in the earth tidal observations. *Sokuchi gakkaishi* (*Journal of the Geodetic Society of Japan*), **6**, 121-135 [In Japanese with English abstract].

Nawa, N. & Miyazaki, K. (2009) The analysis of saltwater intrusion through Komesu underground dam and water quality management for salinity. *Paddy and Water Environment*, **7**, 71-82.

Parker, B. B. (2007) *Tidal Analysis and Prediction*. NOAA Special Publication NOS CO-OPS 3, Silver Spring, Maryland, USA, pp.378.

Pertzev, B. P. (1957) On the calculation of the drift curve in observations of bodily tides. *Bulletin d'Informations Marées Terrestres*, **5**, 71-72.

Sánchez Úbeda, J. P. et al. (2015) Filtrado de datos de niveles piezométricos en acuíferos costeros libres mediante procesamiento de señales ondulatorias. *Geogaceta*, **57**, 135-138 [In Spanish with English abstract].

Serfes, M. E. (1991) Determining the mean hydraulic gradient of ground water affected by tidal fluctuations. *Ground Water*, **29**, 549-555.

Shirahata, K. et al. (2014) New simple method for estimating hydraulic properties of a freshwater-lens aquifer by analysis of tidal groundwater fluctuations. *Noson kogaku kenkyujo giho* (*Technical Report of the National Institute for Rural Engineering*), **215**, 141-154 [In Japanese with English summary].

Shirahata, K. et al. (2016) Improvements in a simple harmonic analysis of groundwater time series based on error analysis on simulated data of specified lengths. *Paddy and Water Environment*. Doi:10.1007/s10333-016-0525-3.

Thompson, R. O. R. Y. (1983) Low-pass filters to suppress inertial and tidal frequencies. *Journal of Physical Oceanography*, **13**, 1077-1083.

Vacher, H. L. (1978) Hydrology of small oceanic islands—influence of atmospheric pressure on the water table. *Ground Water*, **16**, 417-423.

Van der Velde, M. et al. (2007) Sustainable development in small island developing states: Agricultural intensification, economic development, and freshwater resources management on the coral atoll of Tongatapu. *Ecological Economics*, **61**, 456-468.

Yoshimoto, S. (2011) Groundwater flow and transport and potential sources of groundwater nitrates in the Ryukyu Limestone as a mixed flow aquifer in Okinawa Island, Japan. *Paddy and Water Environment*, **9**, 367-384.